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MULTICRITERION LINEAR PROGRAMMING PROBLEMS (Analytical Survey) *

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An analytical survey of man-machine interactive procedures is presented. A decision maker's capabilities to carry out some information processing operations are discussed. A list of a decision maker's elementary operations used in different interactive procedures is given and their estimations from a psychological point of view are suggested. Three main criteria for estimation of man-machine procedures are proposed: reliability of information elicitation from decision maker, insignificant sensitivity to random decision maker's errors, and good speed of convergence to solution. Some procedures are evaluated in terms of these criteria.

1. Introduction

Multicriterion linear programming problems (MLPP) have received increasing attention of many researchers during the last 10-15 years. It is no accident as the problems are rather widespread: they occur in validating economic, organizational and engineering decisions. So, many (usually conflicting) requirements are simultaneously set to the quality of economic problem solutions. For example, a production plan is evaluated by criteria such as profit, cost, resource supply, productivity, and regularity. Along with economic factors there are others that gain in significance: environmental impact, social effect, etc. Clearly, only the account of a variety of criteria is conducive to a rational validation of important national economic decisions. A decision maker (DM) in such problems is usually a manager who formulates the problem and is held responsible for its solution.

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The MLPP have been successfully solved in recent years by identifying the DM's preferences and analyzing the admissible set of decisions. The approach is realized with man–machine procedures (MMP), also referred to as interactive procedures. The multicriteria linear programming problems have to be solved in quite different areas such as: planning of economic development of a region or country (Despontin et al. 1980), planning of the enterprise output (Polyakov 1977), university planning (Geoffrion et al. 1972), and allocation of water resources (Haimes and Hall 1974).

Thus, one of the studies (Steuer and Oliver 1976) considers a problem of scarce resource allocation to different strategies of product advertising. There are different methods of advertising, through newspapers, TV, local radio, posters, etc. The cost of each method, depending on its scale, is known; there is an estimated number of potential buyers affected by some or other methods. Four criteria of advertising strategy appraisal are chosen: bringing the information on the product to the consumer, bringing the knowledge of the product to the consumer, bringing the information on the product preference to the consumer, and dissemination of information on the sold product. The model of the problem considered in the article is a model of four criteria linear programming.

The decision MMPs may be treated as a cyclical process of man (DM)–computer interaction. A cycle consists of an analysis and decision-making phase (problem statement for computers) performed by DM, and an optimization phase (search for decision and computation of its characteristics) performed by the computer. During interaction DM identifies problem specifics and comes to realize the necessity of a tradeoff between different criterion values, specifies his preferences and, following the analysis of the suggested decision, supplies additional information which is behind a more sophisticated computer-generated decision. Thus, proceeding from iteration to iteration DM looks for a better decision simultaneously analyzing the objective characteristics of the problem, cross-impact of criteria. This is what lies behind the potential efficiency of interactive decision systems. The search terminates with the computer generating a decision acceptable for DM, and when DM realizes the inexpediency of the further attempts to improve it for the given model.

The key questions concerning the MMP construction – how to distribute functions between man and machine, and how to arrange their interaction – are generally answered in the most general way: each procedure participant should be assigned those operations he performs best of all. It is rather difficult to implement such a recommendation, and it is an independent complex problem. Its solution requires the analysis of the problem area and the human characteristics as applied to the decision process, in particular, his abilities to process multiple factor information. With this in mind, we shall consider a large number of decision MMPs designed for multicriterion linear programming problems with continuous criterion scales.

There are many MMPs of this kind at present. Most fully they are presented in Thiriez and Zionts (1976), Zeleny (1976), Zionts (1977), Fandel and Gal (1980), Grauer and Wierzbicki (1984), Steuer (1986), Zeleny (1982) and in an overview by Hwang and Masud (1979). The first attempt to classify different MMPs by the type of man–machine interaction was a survey by Larichev and Polyakov (1980).

2. Considered problems

The majority of considered MMPs are applied to MLPPs.

Find vector $\bar{x}^* = (x_1, \ldots, x_n)$ lying within the domain:

$$D = \left\{ A\bar{x} \leqslant \bar{b},\ x_i \geqslant 0,\ i = 1, \ldots, n \right\}, \tag{1}$$

where A is $p \times n$ matrix, $\bar{b}$ is p-vector, and maximizing the set of objective functions

$$C_1(x), \ldots, C_m(x), \tag{2}$$

under the most preferable ratio between their values in the decision point. In other words, in a set of P effective (Pareto optimal) solutions of the considered problem one should find solution $\bar{x}^*$ corresponding to the extremum of an a priori unknown DM utility function

$$\bar{x}^* = \arg \max_{x \in D} U(Z(x)), \tag{3}$$

where $\bar{Z} = (C_1(x), \ldots, C_m(x))$.

Many MMPs use three more definitions: weights of criteria $\bar{\lambda}$, goals $\bar{\alpha}$ (desirable values of criteria), thresholds $\bar{l}$ (restrictions on satisfactory values of criteria).

3. General pattern of MMPs

As is seen from the study (Larichev and Polyakov 1980), despite the seeming difference between various MMPs one may distinguish the common stages and break them down into separate steps.

Each MMP consists of alternating phases of analysis and optimization. The phase may involve several steps.

Optimization phase

Computer:

(A) by making use of the information I_{DM}^{i-1}, elicited from DM, transforms, if necessary, the allowed solution domain D^i and/or exercises additional computations of the objective function $C_k(x)$, $k = 1, \ldots, m$ parameters, say, weights;

(B) computes, on the basis of new data, $\bar{x}^i$ and Z^i;

(C) generates auxiliary information I_{COM}^i.

Analysis phase

Decision maker:

(D) evaluates the offered solution $\bar{x}^i$ (or several solutions) and determines whether it is acceptable. If yes, then the procedure is over, otherwise he analyzes the auxiliary information I_{COM}^i;

(E) feeds additional information I_{DM}^i in the computer making it possible to compute a new solution $\bar{x}^{i+1}$ at the next interaction.

The MMPs developed for solution of the above problems differ from one another in content and implementation of the aforementioned steps. The procedure efficiency largely depends on the nature of man–machine interaction expressed in terms of I_{DM}, I_{COM} amount and quality.

Consider a number of MMPs from this standpoint. With respect to interaction they may be classified into three groups: direct MMPs, MMPs of vector criterion, and MMPs of search for satisfactory criterion values.

4. Direct MMP

Characteristic of this type of procedures is that man is directly engaged in a search for a preferable solution, assigning at each step a new solution $\bar{x}$ or new values to variables (weights $\bar{\lambda}$, goals $\bar{\alpha}$, and thresholds $\bar{l}$) by which it can be computed. Thus, the direct procedure lacks step A. The given approach to the MMP arrangement proceeds from an assumption that DM easily finds a necessary tradeoff between criterion values, that intuition and experience help him act correctly in the routine environment following the analysis of the domain.

A characteristic example of a direct procedure is provided by SIGMOP (Monarchi et al. 1976).

First, the problem is normalized. DM assigns sampling initial values to vector weights $\bar{\lambda}^0$ and thresholds $\bar{l}^0$ as well as to the vector of marginal relative variances $\bar{q}^0$ which are used on constructing domain D.

Step B. Solution is computed.

Step C. Information I^i_{COM} is formed only to unsatisfactory criterion values of solution $\bar{x}^i$, i.e. the extreme solutions are computed only for those objective functions $C_k(x)$ which do not meet the requirement $C_k(\bar{x}) \geqslant \bar{l}^i_k$.

Step D. Decision maker assigns new values to vectors $\bar{\lambda}^{i+1}$, $\bar{l}^{i+1}$ and, if necessary, changes vector $\bar{q}^i$.

According to the authors, the suggested procedure allows DM to operate in the most natural way separating the desired from the required, consistently accomplishing the goals starting with the most significant one (goals ranking may vary from iteration to iteration).

5. MMP of vector criterion estimation

Here DM directly assesses the alternative preferability in a multidimensional criterion space.

A characteristic example is provided by the GDF procedure (Geoffrion, Dyer and Feinberg 1972). In it DM determines the size and direction of steps in the criterion space that ensure the maximum increment of the utility function. The procedure is based on an assumption that DM can specify such increment δ_j of any criterion $C_j(x)$, $j \neq k$ for any point $\bar{Z}$ of criterion space that will be completely compensated by a unit reduction in the value of the reference criterion $C_k(x)$. The thus obtained vector of marginal rates of substitution (MRS) determines the DM utility function gradient in point $\overline{Z}$.

Below follows the MMP GDF:

Step B. Determine $\bar{x}^i$, $\overline{Z}^i$.

Step D. DM analyzes $\overline{Z}^i$. Information I^i_{DM} is collected in the process of interaction with the computer involving the following actions (steps of A and E type):

(a) DM determines the MRS in the decision point $\overline{Z}^i$.
(b) The MRS determine the 'best' direction in the decision space along which the DM utility function increases to the maximum extent.
(c) The step size is determined along the assigned direction. The results in the form of a table are presented to DM.
(d) Analyzing the table. DM chooses the point corresponding, in his opinion, to the maximum increase in the utility function. Thereby a new point $\bar{x}^{i+1}$ is determined.

Decision theorists questioned the DM ability to successfully perform the functions assigned by the GDF procedure. It was noted, in particular, that while working with small increments in objective functions, DM will make errors in determining the utility function gradient.

6. MMP of search for satisfactory criterion values

In these procedures DM imposes and changes thresholds $\bar{l}^i_k$ on criterion values in the solution point. First to realize this approach was the MMP STEM (Benayoun et al. 1971).

Step A1. A system of weights $\bar{\lambda}^i$ corresponding to the largest sum of relative criterion values is determined.

Step B. Solution $\bar{x}^i$ and vector $\overline{Z}^i$ are computed.

Step C. Information I^i_{COM} is formed which gives $\overline{Z}^i$ and maximum available values of criteria.

Step D. DM evaluates decision $\bar{x}^i$ and determines whether it is acceptable.

Step E. If no, then DM indicates the values of the last satisfactory criterion, and to what extent it is desirable to improve it, i.e. thresholds l_μ.

Step A2. By adding eq. $C_\mu(x) \geqslant l_\mu$ to domain D^i, a new domain D^{i+1} of admissible solutions is determined.

7. On MMP comparison criteria

The development of a large number of decision MMPs has rightfully raised the question of their comparison. The first attempt was made by Dyer (1973). Nine students acted as decision makers. A model situation was offered to them: assessment of different types of cars by three parameters: price, engine power, and petrol consumption per 100 km. The relationships between the parameters was described with linear equations. The subjects were not acquainted with the direct trial and error method, nor with the GDF method. They were also requested to suggest any other techniques suitable for solution of the given problem. The subjects' assessment (by a five-point scale) of the method effectiveness and the degree of its credibility were used as the methods' assessment criteria. As regards the group as a whole and each subject separately, the GDF method ranked first by these criteria. Note that one of the subjects suggested a method of stepwise minimization of individual criteria which Dyer defined as close to the STEM method.

The second and more serious work on comparing MMPs of multicriteria linear programming problem solution was done by Wallenius (1975). The evaluation was conducted with two groups of subjects: 18 students and 18 managers. On a model example (enterprise management problem) they had to compare three methods: direct trial and error method, the STEM method, and the GDF method. The following criteria of MMP evaluation were selected: (1) subjective ranking of the methods by the subjects; (2) degree of the acquired solution credibility; (3) simplicity of use; (4) ease of the method perception; (5) obtained information utility for DM; (6) number of iterations; (7) solution proximity to extremum. As a result of comparison by the first five criteria, the GDF method turned out to be the worst. By the two principal (last) criteria the STEM method happened to be the best. The conflicting results of the methods comparison testify to the absence of generally accepted guidelines of such a comparison. The employed criteria of method evaluation do not properly describe the possibility of their application to the solution of significant practical problems.

As regards MMPs, one must keep in mind that they are procedures of man and computer interaction. The latter's success depends on human abilities in information processing.

Numerous psychological studies of recent years indicate the highly limited capabilities of humans in processing complex multiattribute information. The results of the research are analyzed in an overview by Kahneman et al. (1982).

The state of affairs can be briefly characterized as follows. Many problems concerned with information processing (comparison of multicriteria objects, ranking of a group of objects, assignment of criteria weights, etc.), on a certain number and nature of criteria, and type of scales become too complex for human beings. In handling such problems people make errors, become inconsistent, and use auxiliary heuristics for simplification of the problem. Such behavior often remains unnoticed due to difficulties in its study, the flexibility of man, and his ability to adapt a problem to his capabilities. In any information processing problem there are certain human 'capability limits' characterized by the number of criteria, types of scales, and requirements to the final type of solution. It should be noted that within the capabilities and beyond them, human behavior differs considerably (Larichev and Moshkovich 1980).

It is important to emphasize that the errors in human responses that cause concern, are not insignificant. Beyond the 'human capabilities' people behave quite differently. Thus, for example, they stop using a considerable part of information, and break the condition of transitivity under some circumstances.

Such behavior makes it necessary to look differently at the requirements set to DM within the frameworks of MMP. The actions required from DM may be consistent with human capabilities in information processing or be too complex for him/her.

Hence, the first criterion of the decision method estimation is the consistency of the method requirements with the possibilities of eliciting reliable information from people. The second criterion is the MMP stability to random DM errors. The third criterion is the presence of MMP convergence and of satisfactory convergence speed.

These are, we believe, three basic criteria. They determine the estimates by such secondary criteria as the degree of MMP credibility, utility of the supplied information, etc.

To evaluate MMP from this angle, it is necessary to thoroughly analyze the decision-making phase with a view to singling out simple information processing operations. Then assess DM's abilities in accomplishing these operations. This will make it possible to judge the correctness of MMP as a whole.

8. Analysis of elementary operations

All operations in MMP on information processing by man (decision maker) can be classified into four groups: operations with criteria specification; operations with individual criterion values of one alternative; operations with alternative as a set of values of all criteria; and operations with variables. The majority of MMPs make use of the first three groups of operations.

An operation is referred to as elementary if it cannot be broken down into other operations relating to objects of the same group.

In evaluating an operation, account should be taken of the problem parameters: number of criteria, character of scales, number of quality divisions on discrete scales, number of variables, quantity of alternatives, and nature of alternative estimates by criterion scales (quantitative, qualitative, approximate, accurate). Clearly, the fewer alternatives, criteria, variables, and scale divisions, the simpler an operation. The elementary operations can be defined as:

– complex (C) if the psychological research indicates that in performing these operations DM is often inconsistent and/or uses simplified strategies (e.g., drops some criteria);

– complex except for small problems (CS) if the psychological research indicates that in performing these operations for small problems (2–3 criteria, 2–3 alternatives) DM does it with inconsiderable contradictions but for bigger problems DM is often inconsistent and/or uses simplified strategies;

– admissible (A) if the research indicates that DM can perform them with inconsiderable contradictions and complex strategies (e.g., estimating a set of several criteria values). As a rule, there are standard, routine operations for DM;

– uncertain (U, UC, UA) if no research has been carried out on these operations, but by analogy one may judge about the operation admissibility (UA) or complexity (UC).

01. Operations with criteria specification

Operation 011. Assignment of criterion weights.

This operation is used very often. At the same time the reliability of weights assigned by the subject (DM) is highly doubtful. It follows from the experimental results that man makes considerable errors in assigning the criterion weights as compared with the objectively known ones, that the weights contradict his immediate alternative estimates, etc. (Kahneman et al. 1982; Slovic and Lichtenstein 1971). Though the discussion on the expediency of the weights' use in decision methods is still going on, the obtained data are sufficient to consider this operation as rather complex for DM (Stewart and Ely 1984).

Operation 012. Criteria ordering by significance.

There are few studies that conducted a thorough analysis of this method of information elicitation from DM. The results of the experiment (Larichev et al. 1980) under seven criteria and two divisions on criterion scales can be viewed as positive. Besides, a special experiment was carried out (Nikiforov et al. 1984), which made it possible to conclude that criteria, meaningful for DM, are reliably and consistently ranked. As a whole this operation can be described as A.

02. Operations with individual criterion values of one alternative

Operation 021. Comparison of two values by one criterion scale.

With interval and ordinal scales, both continuous and discrete, this operation is very simple, hence estimated as A.

Operation 022. Comparison of values of two different criteria (or differences on two criteria scales) under fixed values of other criteria. Such comparison aims at identifying preferability of one of the values of their equivalence.

The research we conducted under 8 criteria indicates that DM reliably performs this operation with a small number of contradictions. Finding a tradeoff between the values of two conflicting criteria is a typical operation conducted by DM in different decision-making situations. On the whole, this operation may be referred to as A.

Operation 023. Quantitative determination of changes in the value of one criterion equivalent to the assigned changes in the value of the other.

The quantitative determination of the unity function gradient is rather difficult for DM. Note, in the first place, that equal changes are concerned here. Besides, determination of quantitative values of utility is difficult for DM (Tversky 1969). On the whole, the operation may be referred to as UC.

Operation 024. Determination of a satisfactory value by one criterion.

There has been no systematic research into this operation. According to numerous papers of descriptive character the transference of a criterion into a constraint, search for a satisfactory level is a typical operation human beings employ in different problems. Some errors may occur due to the phenomenon 'anchoring' though there are methods for preventing such errors (Kahneman et al. 1982). On the whole, the operation can be defined as UA.

Some operations from the considered group 02, used in different MMPs, are not elementary and can be decomposed into elementary operations. For example:

Operation 025. Identification of all or part of criteria whose values must be improved, may deteriorate, or must remain not worse than the attained satisfactory level.

This elementary operation can be represented by a combination of operations 021 and 022. On the whole, it can be defined as A.

Operation 026. Identification of criteria whose values are most unsatisfactory (satisfactory).

This operation can be presented as a set of operation 022 and defined as A.

03. Operations with alternatives as a set of values of all criteria

Operation 031. Comparison of two alternatives and identification of the best (worst) one.

There is numerous systematic research into this way of information elicitation (Tversky 1972, 1969). The results show that this operation is very difficult for DM, especially on a large number of criteria. Already at more than three criteria use is made of simplified heuristics that can lead to inconsistencies (Russo and Rosen 1975). On the whole, the operation can be defined as CS.

Here are several more rather widespread operations which are not elementary.

Operation 032. Choice of the best (worst) alternative.

According to the best descriptive research (Kahneman et al. 1982) it can be presented as determination of the order of comparison followed by operation 031. On the whole, it can be treated as CS.

Operation 033. Determination of an 'ideal' alternative proximity which determines the quality of the current solution.

The operation can be presented as a combination of operation 031, i.e., comparison of the alternative with the best one, each value of which was obtained by defining the maximum by the respective criterion. Aggregate estimate in CS as it incorporates operation 031 with estimates CS.

04. Operations with variables

Operation 041. Identification of variables that must be increased (reduced).

Though no systematic study has been carried out, this operation looks very complex for DM. First of all, variables x do not show an evident trend toward the desirable changes like criteria. Besides, the number of these variables generally exceed the number of criteria. The general estimate of the operation is UC.

The aggregate table 1 contains estimates of the operations.

We would like to conclude with a general remark. If any parameter (criterion weight, alternative rank, etc.) is assigned a range of its possible values, then such information is much more simple for DM though the total estimate of an elementary operation is likely to remain intact. In assigning the range boundaries a problem arises, close to the basic one by complexity.

Table 1

Operation number	Name of operation	General estimate
011	Assignment of criteria weights	C
012	Criteria ordering by significance	A
021	Comparison of two values on a single criterion scale	A
022	Comparison of two criteria values variation	A
023	Quantitative definition of a criterion value variation that is equivalent to variation of another criterion value	UC
024	Assignment of satisfactory value by single criterion	UA
025	Identification of those criteria whose values must be improved, may deteriorate, remain at least equal to the attained satisfactory level	A
026	Differentiation of criteria values unsatisfactory (satisfactory) to the greatest extent	A
031	Comparison of two alternatives and identification of the better one	CS
032	Choice of the best (worst) alternative from a set	CS
033	Determination of an 'ideal' alternative proximity the degree of which determines the quality of the current solution	CS
041	Identification of variables	UC

9. Analysis of MMP correctness

It seems desirable to evaluate MMPs by the following two criteria: (1) If an MMP incorporates A or UA operations relating to information elicitation from DM, then it is superior to MMPs using operations

Table 2

No. of elem. oper.	Estimate	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
011	C												*							
012	UA														*					
022	A	*	*																	
023	UC								*									*	*	
024	UA	*	*				*						*							
025	A					*		*												
026	A	*	*	*	*		*													
031	C										*	*		*				*		
032	C								*					*	*	*	*			
033	C												*							*
041	C									*										
Sensitivity to DM's answers		S	S	S	S	S	S	G	S	S	S	G	S	G	G	G	S	G	S	S
General estimate		C	C	C	C	C	C	IC	IC	IC	IC	IC	IC	IC	IC	IC	IC	IC	IC	IC

* Designates accomplishment of the given operation.

with estimates C, CS and UC. (2) As is known, DM can make errors at any information processing operation. The analysis of different MMPs shows that they feature different sensitivity to DM's errors. An MMP in which a random DM error may completely eliminate from consideration a part of admissible solution domain, containing a more preferable solution x, can be considered as having greater sensitivity to random errors of DM (G). In other MMPs, a random error results just in an increased number of iterations. Consider them as having smaller sensitivity to random DM errors (S). This criterion is important because in many practical cases DM has one interactive session for defining a compromise between criteria and finding the best solution.

Table 2 contains several MMPs with their estimates by the two aforementioned criteria. A general estimate of the procedure correctness (C – correct; IC – incorrect) is determined on the basis of the positive estimates by two criteria: correctness of the employed operations and insignificant sensitivity to DM random errors.

The table considers the following MMPs: (1) STEM (Benayoun et al. 1971); (2) IMGP (Spronk 1980); (3) BK (Belenson and Kapur 1973); (4) E.B.A. (Aubin and Naslund 1972); (5) (Michalowski and Zolkiewski 1982); (6) (Benson 1975); (7) GCOM (Fandel 1977); (8) GDF (Geoffrion et al. 1972); (9) (Savir 1966); (10) P.O.P. (Benayoun and Tergny 1970); (11) ZW (Zionts and Wallenius 1976); (12) SIGMOP (Monarchi et al. 1976); (13) SIMOLP (Reeves and Franz 1982); (14) HOPE (Ho 1979); (15) IMOLP (Steuer 1977); (16) DI (Zeleny 1974); (17) (Musselman and Talavage 1980); (18) ISWT (Chankong and Haimes 1977); (19) (Wierzbicki 1980).

From table 2 we may draw several conclusions.

(1) Only the first six out of the 19 considered methods can be treated as correct (C). The rest of the methods either employ complex (or supposedly complex) operations, or are too sensitive to random DM errors.

(2) The decomposition of the analysis phase into information processing operations conducted by DM reveals the identity of the whole number of methods. Thus, the IMGP method, as regards information elicitation from DM, does not differ from the earlier suggested STEM method. It should be noted that the correct MMPs use only operations from group 02.

(3) The majority of incorrect methods most often use operations 031 and 032. Generally speaking, in these methods use is made of a wider range of operations – from all four groups.

(4) Table 2 does not contain direct MMPs, apart from SIGMOP. All such methods use operation 011 which is difficult for DM.

Pass now to the third criterion of MMP evaluation – convergence to the domain containing an extreme value. We presume that while analyzing the two presented solutions DM chooses the one with the largest utility. The difference between correct and incorrect procedures, from this point of view, consists in the reliability of comparison and choice operations. The operations performed by DM within the frameworks of correct procedures are much more reliable. Consider six correct MMPs regarding the convergence speed.

Take procedures 3 and 4 in table 2, in which minimal information is elicited from DM. After DM indicates criteria whose estimates should be changed either way, the computer performs an optimization phase and finds a new solution. Clearly, this solution will be at the boundary of the allowed domain. In a general case, it does not satisfy DM and the cycle is repeated.

The specifics of these MMPs are that DM does not control the subsequent solution point – it is simply computed by the machine. If $U^i(Z)$ is the value of the DM utility function at the i-th iteration, then there is no guarantee that $U^{i+1}(Z) > U^i(Z)$. It is possible to select the domain in such a way that the cycling occurs, i.e., at the i-th iteration DM refers to the values by one group of criteria, as unsatisfactory, and as satisfactory by another, and at the $(i+1)$ iteration DM's information is opposite.

Consider now procedures 1, 2, 5, 6. Here at the analysis phase, DM compares the improvement by one criterion and deterioration by a possible extreme value of another. The acquired tradeoff, i.e. the value of $U^i(Z)$ is fixed. The following iteration compares exactly with this value, which makes it possible to guarantee $U^{i+1}(Z) > U^i(Z)$. In practical applications of the STEM method, the tradeoff by each pair of criteria was accomplished for 2–3 steps and was fixed by the imposition of constraints on the criterion values. Hence, the extremum domain was gained for $2m$–$3m$ queries to DM which can be considered a rather good speed of convergence. It may be concluded that the pairwise determination of a tradeoff between criteria is, on the one hand, a correct procedure and, on the other, provides for the MMP convergence to the extreme value.

Note that it is a rather approximate conclusion requiring additional analysis of the rate of correct MMP convergence.

10. Conclusion

The requirement of MMP correctness is, we believe, necessary for its scientific substantiation and successful application. This does not rule out, however, that under 2–3 criteria direct methods may turn out to be an efficient tool in the hands of a skilled DM. Under more than three criteria (a continuous domain of admissible values) the number of possible DM errors increases and direct methods become unreliable.

In the light of the above analysis, it is hardly reasonable to employ MMPs of vector criterion value assessment. In complex problems they are inferior to MMPs of search for satisfactory criteria values, and in simple problems it would be more logical to make use of direct MMPs. This is also confirmed by the analysis of practical MMP applications.

At present, a considerable practical need for man–machine procedures of solving multicriteria linear programming problems is evident. To meet this need, it is necessary to use those methods that promise much wider application from the point of view of three basic criteria: reliability of information elicitation from decision makers, insignificant sensitivity to random DM errors, and good speed of convergence to solution.

References

Aubin, J.P. and B. Naslund, 1972. An exterior branching algorithm, WP-72-42. Brussels: European Institute for Advanced Studies in Management.

Belenson, S.M. and K.C. Kapur, 1973. An algorithm for solving multicriterion linear programming problems with example. Operational Research Quarterly 24, 65–77.

Benayoun, R. and J. Tergny, 1970. Mathematical programming with multi-objective function: a solution by P.O.P. (Progressive Orientation Procedure). METRA 9, 279–299.

Benayoun, R., J. Montgolfier, J. Tergny and O. Larichev, 1971. Linear programming with multiple objective functions: STEP method (STEM). Mathematical Programming 1, 366–375.

Benson, R.G., 1975. Interactive multiple criteria optimization using satisfactory goals. Dissertation, University of Iowa.

Chankong, V. and Y.Y. Haimes, 1977. 'Interactive SWT method for multiobjective decision-making'. In: S. Zionts (ed.), Multiple criteria problem solving: proceedings, Buffalo, NY, Vol. 155. Berlin: Springer. pp. 42–67.

Despontin, M., P. Nijkamp and J. Spronk, 1980. Macro-economic planning with conflicting goals. Berlin: Springer.

Dyer, J.S., 1973. 'An empirical investigation of man–machine interactive approach to the solution of multiple criteria problem'. In: J.L. Cohrane and M. Zeleny (eds.), Multiple criteria decision making. Columbia, SC: University of South Carolina Press.

Fandel, G., 1977. 'Public investment decision making with multiple criteria: an example of university planning'. In: S. Zionts (ed.), Multiple criteria problem solving: proceedings, Buffalo, NY, Vol. 155. Berlin: Springer. pp. 116–130.

Fandel, G. and T. Gal (eds.), 1980. Multiple criteria decision making: theory and application, Vol. 177. Berlin: Springer.

Geoffrion, A.M., J.S. Dyer and A. Feinberg, 1972. An interactive approach for multi-criterion optimization with an application to the operation of an academic department. Management Science 19, 357–368.

Grauer, M. and A.P. Wierzbicki (eds.), 1984. Proceedings of an international workshop on interactive decision analysis and interpretative computer intelligence, Vol. 229, IIASA, Laxenburg. Berlin: Springer.

Haimes, Y.Y. and W.A. Hall, 1974. Multiobjectives in water resources systems analysis: the Surrogate Worth trade-off method. Water Resources Research 10, 615–624.

Ho, J.K., 1979. Holistic preference evaluation in multiple criteria optimization. New York: Brookhaven National Laboratory, AMD 25656.

Hwang, C.L. and A.S.M. Masud, 1979. Multiple objective decision making – methods and applications: a state-of-the-art survey, Vol. 164. Berlin: Springer.

Kahneman, D., P. Slovic and A. Tversky (eds.), 1982. Judgement under uncertainty: heuristics and biases. Cambridge: Cambridge University Press.

Larichev, O.I. and H.M. Moshkovich, 1980. 'On elicitation of consistent estimates of multiple criteria alternatives'. In: VNIISI, Transactions, issue 9. Moscow: VNIISI.

Larichev, O.I. and O.A. Polyakov, 1980. Man–computer procedures of multiple objective mathematical programming problem solving. Economika i Matematicheskije Metody 16, 129–145.

Larichev, O.I., V.S. Boychenko, H.M. Moshkovich and L.P. Sheptalova, 1980. Modelling multiattribute information processing strategies in a binary decision task. Organizational Behavior and Human Performance 26, 278–291.

Marschak, J., 1968. Decision making: economic aspects. International Encyclopedia of the Social Sciences 4, 42–55.

Michalowski, W. and Z. Zolkiewski, 1982. Interactive approach to solving of a linear production planning problem with multiple objectives, 5th International Conference on Multiple Criteria Decision Making, Mons, Belgium.

Monarchi, D.E., J.E. Weber and L. Duckstein, 1976. 'An interactive multiple objective decision making aid using nonlinear goal programming'. In: M. Zeleny (ed.), Multiple criteria decision making: Kyoto 1975, Vol. 123. Berlin: Springer. pp. 235–253.

Musselman, K. and J. Talavage, 1980. A tradeoff cut approach to multiple objective optimization. Operations Research 26, 1424–1435.

Nikiforov, A.D., S.B. Rebrik and L.P. Sheptalova, 1984. 'Experimental investigation of stability preference in some problem of decision making'. In: VNIISI, Transactions, issue 9. Moscow: VNIISI.

Polyakov, O.A., 1977. Optimization decision making method for enterprise. Avtomatica i Telemehanika 2, 92–101.

Reeves, G.R. and L.S. Franz, 1982. A simplified interactive multiple objective linear programming procedure, 5th International Conference on Multiple Criteria Decision Making, Mons, Belgium.

Russo, J.E. and L.D. Rosen, 1975. An eye fixation analysis of multialternative choice. Memory and Cognition 3, 267–276.

Savir, D., 1966. Multiobjective linear programming. ORC-66-21. Operations Research Center, University of California, Los Angeles, CA.

Slovic, P. and S. Lichtenstein, 1971. Comparison of Bayesian and regression approaches to the study of information processing in judgement. Organizational Behavior and Human Performance 6, 649–744.

Slovic, P., B. Fischhoff and S. Lichtenstein, 1977. Behavioural decision theory. Annual Review of Psychology 28, 1–39.

Spronk, J., 1980. Interactive multiple goal programming for capital budgeting. Dissertation, Erasmus University, Rotterdam.

Steuer, R.E., 1977. 'An interactive multiple objective linear programming procedure'. In: M.K. Starr and M. Zeleny (eds.), Multiple criteria decision making, New York: Elsevier. pp. 225–239.

Steuer, R.E., 1986. Multiple criteria optimization. New York: Wiley.

Steuer, R.E. and R.L. Oliver, 1976. An application of multiple objective linear programming to Media Selection. OMEGA 4, 455–462.

Stewart, T.R. and D.W. Ely, 1984. Range sensitivity: a necessary condition and a test for the validity of weights. Paper prepared for Multiple Criteria Decision Making Conference. Cleveland, OH.

Thiriez, H. and S. Zionts (eds.), 1976. Multiple criteria decision making: Jouy-en-Josas. Vol. 130. Berlin: Springer.

Tversky, A., 1969. Intransitivity of preferences. Psychological Review 76, 31–48.

Tversky, A., 1972. Elimination by aspects: a theory of choice. Psychological Review 79, 281–299.

Wallenius, J., 1975. Comparative evaluation of some interactive approaches to multicriterion optimization. Management Science 21, 1387–1396.

Wierzbicki, A.P., 1980. 'The use of reference objectives in multiobjective optimization'. In: G. Fandel and T. Gal (eds.), Multiple criteria decision making: theory and application, Vol. 177. Berlin: Springer. pp. 468–486.

Zeleny, M., 1974. A concept of compromise solutions and the method of displaced ideal. Computers and Operations Research 1, 479–496.

Zeleny, M. (ed.), 1976. Multiple criteria decision making: Kyoto 1975, Vol. 123. Berlin: Springer.

Zeleny, M., 1982. Multiple criteria decision making. New York: McGraw-Hill.

Zionts, S. (ed.), 1977. Multiple criteria problem solving: proceedings, Buffalo, NY, Vol. 155. Berlin: Springer.

Zionts, S. and J. Wallenius, 1976. An interactive programming method for solving multiple criteria problem. Management Science 22, 652–663.