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Experiments Comparing Qualitative Approaches to Rank Ordering of Multiattribute Alternatives

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ABSTRACT

ZAPROS, a method to support rank ordering tasks using ordinal input from decision makers, is discussed and compared with a preference cone technique and the analytic hierarchy process (AHP). It provides a means to identify inconsistencies in ordinal decision tasks, yielding verification and explanation of results for partial ordering of a large set of alternatives. The results indicate that ZAPROS provides no less accuracy in task solution, while having some advantages from a behavioural point of view. Comparative analysis of the effectiveness of the methods under consideration in accordance with differences in task characteristics is carried out.

KEY WORDS Analytic hierarchy process Preference cone methods ZAPROS

1. INTRODUCTION

All multiattribute decision methods use decision maker or expert judgment in one way or another. Elicitation of such information is necessary to eliminate the uncertainty connected with decisions in the presence of multiple criteria, to generate necessary compromise and to yield good decisions.

The deep complexity of eliciting information from humans has been noted by many psychologists and researchers in decision making (Slovic *et al.*, 1977; Kahneman *et al.*, 1982; Larichev, 1984). The limitations in human capabilities to evaluate and compare multiattribute options are well known. These limitations can lead to inconsistencies in human judgments (Hoffman *et al.*, 1968; Tversky, 1969; Russo and Rosen, 1975) or to implementation of simplified rules which do not take into account some essential aspects of the options under consideration

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(Payne, 1976; Montgomery, 1977; Larichev and Moshkovich, 1988). These factors must be considered when designing methods to aid decision making. Understanding what information forms are reliable is highly important. It is also necessary to develop means for verification of consistency of information provided by decision makers and to correct detected errors. It would also be useful to have explanations of detected inconsistencies in terms a decision maker would understand.

In Larichev *et al.* (1987) an attempt was made to collect and classify all elementary operations in information processing used in normative decision making. Twenty-three such operations were defined and analysed from the point of view of their complexity for human beings. The primary conclusion of that study was that quantitative evaluation and comparison of different objects are much more difficult for subjects than conducting the same operations using qualitative (ordinal) expressions.

This conclusion is currently popular and is based not only on the data of descriptive investigations but also on experience in real decision problems. Thus methods have been presented which try to use ordinal (verbalized) judgments while working with people (preference cones (Koksalan *et al.*, 1984; Korhonen *et al.*, 1984), ELECTRE (Roy, 1968), artificial intelligence (Hart, 1985) approaches). However, as a rule this information is used to derive numerical values which are used to obtain the decision. In such cases the result is hardly independent of the accepted approach to scaling of the obtained judgments.

ZAPROS (Larichev and Moshkovich, 1991) is a method developed to aid in qualitative evaluation of multiattribute alternatives. It not only elicits information from a decision maker in a qualitative form but tries to use it without resort to numbers and to apply rational logic to categorize a database of alternatives into categorical levels. Human preferences are obtained interactively. Logical inconsistencies can be identified and the decision maker prompted for clarification in such instances. In this paper we compare ZAPROS with the well-known preference cone and analytic hierarchy process (AHP) methods.

Section 2 of this paper presents the main ideas of the ZAPROS method. Section 3 considers the experimental design of preliminary tests for comparing ZAPROS with a preference cone system and AHP. The results of the tests are given in Section 4. Section 5 contains a discussion of the test results and Section 6 presents conclusions.

2. THE MAIN IDEAS OF THE ZAPROS SYSTEM

2.1. Task formulation

A number of decision problems involve a rather large but finite number of alternatives estimated upon a set of criteria. Examples of such decision problems are common (to choose a house, buy a car, select a person for a position, select a job). In such problems people tend first to choose a small subset of alternatives potentially attractive for them, which later are considered more thoroughly in making the final decision (e.g. Roy, 1968; Payne, 1976; Korhonen *et al.*, 1990). In other cases it may be enough to partially rank order alternatives (e.g. formation of R&D plans on the basis of proposals (Larichev, 1982) or portfolio selection (Clarckson, 1962)). In these cases we need to have a partial ordering of alternatives in order to fund as many of the best projects as we can.

In cases when we have too many alternatives (maybe hundreds), it may be considered logical enough to construct some rule for pairwise comparison of alternatives on the basis of decision maker (DM) preferences in the criteria space and to use this set of rules for rank ordering of the set of real alternatives. The method we present supports decisions where criterion scales are discrete and have verbal (qualitative) formulations and need to be rank ordered from the best to the worst (see Appendix I for example). The problem may be stated as follows.

Given:

(1) a set of criteria $K=\{q_i\}$, $i=1,2,\ldots,Q$;
(2) n_i, the number of possible values on the scale of the ith criterion ($i \in K$);
(3) X_{ij}, the scale of the ith criterion with j values ordered from the best to the worst;
(4) Y, a set of vectors $y_i \in Y$ of the type $y_i=(y_{i1}, y_{i2},\ldots,y_{ij})$;
(5) $A=\{a_i\}$, a set of vectors describing the alternatives.

Required: to form an ordering of multiattribute alternatives of the set A on the basis of the decision maker's preferences.

The main idea of the approach described below is based on the concept of joint ordinal scale built according to the DM's preferences. The joint ordinal scale (JOS) means that all possible values upon all criteria are rank ordered for the DM in accordance with his or her preferences. This ordinal scale may be effectively used for comparison of real alternatives. Implementation is based on two rather simple assumptions about the properties of a decision maker's preference system: preferential independence of criteria (Fishburn, 1970; Keeney and Raiffa, 1976) and transitivity of the resulting preference–indifference relation (Mirkin, 1974). The main stages in the process of task solution are:

(1) elicitation of information on DM preferences;
(2) elimination of inconsistencies in the information about DM preferences;
(3) construction of a joint ordinal scale;
(4) implementation of the JOS to compare real alternatives.

2.2. Procedure for elicitation of decision maker preferences

To construct the joint ordinal scale, it is necessary to compare pairs of different values upon different criteria. Since the values upon other criteria may influence the comparison result, we propose to compare hypothetical alternatives (vectors from Y) which have all the same values but two. The number of such pairs from Y is very large. Therefore it was proposed to compare vectors near the reference point, since it will be shown that this information is sufficient to construct a joint ordinal scale.

Definition 1

The reference point is the vector with all the best values on all criteria.

Definition 2

L is a list of vectors near the reference point if L is a subset of vectors from Y with all components except one equal to those of the reference point.

Thus $L=\{y_i \in Y | y_i=(1,1,1,\ldots,1,y_{is},1,\ldots 1), y_{is} \neq 1, \forall s \in K\}$, where 1 indicates the most preferable value upon the corresponding criterion from K.

We assume that each of the criteria have a finite number of ordered values. It is therefore evident that there is a finite number of vectors. Some dominance relationships among these vectors are identifiable immediately. Vector y_i is preferred to vector y_j if performance on all criteria for y_i is at least as good as that of y_j for all criteria and for at least one criterion the performance of y_i is better than that of y_j. Some pairs of vectors from $L \times L$ may be compared beforehand on the basis of such binary relations.

The DM is asked to compare vectors differing on only one attribute. All questions necessary to compare all pairs of vectors from the list L are asked. The results of pairwise comparisons by a DM may be presented in the form of binary relations as follows:

(1) P_{DM} is the list of DM preferences between pairs of vectors;
(2) I_{DM} is the list of pairs of vectors between which the DM is indifferent.

If we require the transitivity of relations P_{DM} and I_{DM}, then according to Mirkin (1974), the relationship $R = P_{DM} \cup I_{DM}$ is a linear quasi-order on the set L.

In any interview with a DM there is a possibility of errors in his or her responses. These errors may be random or may occur while comparing alternatives similar in quality. Therefore inconsistencies (contradictions) may appear in the information elicited from a DM. A special procedure for detection and elimination of contradictions in DM responses is proposed.

2.3. Elimination of intransitivity in decision maker responses

Possible contradictions in DM responses in our case may be determined as violations of transitivity of the relations P_{DM} and I_{DM}. In general the problem of detection and elimination of intransitivity in pairwise comparisons is rather complicated. It is analogous to the task of cycle elimination in a graph (Wilson, 1972; Kendall, 1969; Aho *et al.*, 1974; Ore, 1962). Figure 1 demonstrates a very simple intransitive set of preferences among three choices, along with three possible changes to resolve the intransitivity. It is known that the problem of determination of the minimal number of arcs necessary to be eliminated from a graph to make it acyclic is an NP-complete problem (Garey and Johnson, 1979). This means that solution of this problem cannot be guaranteed in polynomial time. That is why there are a number of works devoted to the development of an approximate solution of this problem (Aho *et al.*, 1974; Ore, 1962).

In detection and elimination of intransitivities in the information received from a DM, there are two peculiarities which make the traditional approach mentioned above ineffective. First of all, the elimination of arcs in a graph may lead to a partial loss of information on vector comparisons, which is undesirable. Secondly, our task consists not of determination of the minimal number of arcs to be eliminated, but detection of the erroneous DM responses which have led to the cycles. The main idea of the approach adopted in ZAPROS (Moshkovich, 1988) is as follows. Since the transitivity of DM preferences is initially assumed and violations of

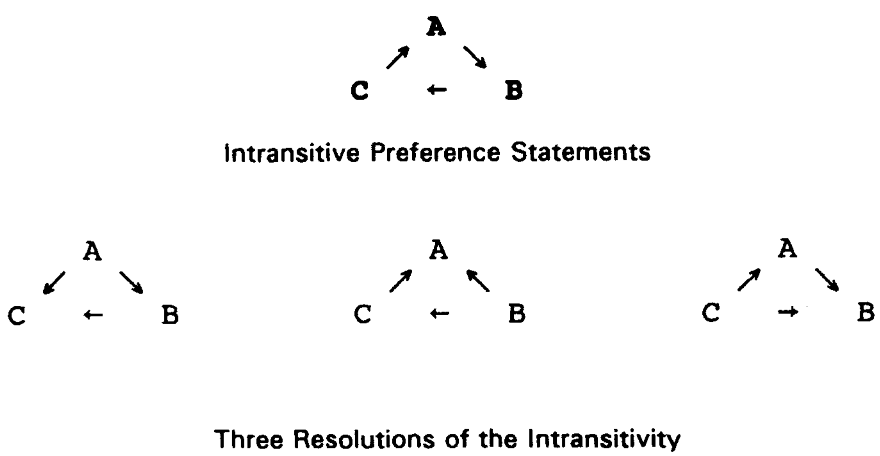

Figure 1. Some possible resolutions of intransitivity

this assumption are considered to be errors in DM responses, we suggest the following procedure. After each comparison of vectors from L made by a DM, this information is extended on the basis of transitivity (transitive closure of the binary relation defined on the set L is being built). Let us assume that a DM has been presented with vectors $y_i \in L$ and $y_j \in L$ and has responded that y_i is more preferable than y_j. This means that $(y_i, y_j) \in P_{DM}$. Then for any y_k such that $(y_j, y_k) \in P_{DM}$ or $(y_j, y_k) \in I_{DM}$ it is possible to say that $(y_i, y_k) \in P_{DM}$.

If a DM has responded that y_i is equal to y_j (and this means that $(y_i, y_j) \in I_{DM}$), then for any $y_k \in L$ such that $(y_j, y_k) \in I_{DM}$ it is possible to say that $(y_i, y_k) \in I_{DM}$ and for $\forall y_k \in L$ such that $(y_j, y_k) \in P_{DM}$ it is possible to say that $(y_i, y_k) \in P_{DM}$. After that evaluation the DM is presented with the next pair of vectors from L for which the relation has not been defined. When the DM's response is obtained, transitive closure is developed and maintained up to the moment of establishing relationships for all pairs of vectors from L. It is necessary to note that during this procedure, violations of transitivity cannot occur. This is evident because the DM is presented only with pairs of vectors from $L \times L$ for which previous responses have not predefined any relation. Thus any variant of the response (for comparison of these vectors) will not contradict previous responses. Once the response is received, transitive closure of the newly obtained relation is being built. It is known that transitive closure of the acyclic graph does not lead to cycles (Aho *et al.*, 1974), so we can say that such a procedure does not lead to intransitivity of the relation being built. To test responses given by a decision maker, the DM is presented with additional pairs of vectors for comparison on the basis of the following principle: the relation between each pair of vectors from L is to be defined directly (by the DM's response) or indirectly (by transitive closure) no less than two times. This requirement means that if the DM verifies the relation between some pair of vectors from L, either directly or indirectly through transitive closure, then this relation is considered to be proven. If the relation between some pair of vectors from L has been defined only once and only by transitive closure, then this pair of vectors is presented to the DM for comparison.

If the DM's response does not conflict with the previously obtained information, then the judgment is considered to be correct. If there is some difference, we find the triple of vectors for which a pairwise comparison contradicts the transitivity of the relation being built on L, i.e. of vectors y_i, y_j, $y_k \in L$ such that one of the following statements is fulfilled:

(1) $(y_i, y_j) \in P_{DM}$, $(y_j, y_k) \in I_{DM}$, $(y_i, y_k) \in I_{DM}$;
(2) $(y_i, y_j) \in P_{DM}$, $(y_j, y_k) \in I_{DM}$, $(y_k, y_i) \in P_{DM}$;
(3) $(y_i, y_j) \in P_{DM}$, $(y_j, y_k) \in P_{DM}$, $(y_i, y_k) \in I_{DM}$;
(4) $(y_i, y_j) \in P_{DM}$, $(y_j, y_k) \in P_{DM}$, $(y_k, y_i) \in P_{DM}$.

Such a triple may always be detected, because after each of the DM's responses we have built transitive closure of the obtained relation. In this case the DM is asked to reconsider the situation and to change one or more of his or her responses to eliminate intransitivity.

After the corrected responses are obtained, they are incorporated into the information on the DM's preferences as follows. It is supposed that when we start the interview with a DM, we have only the three responses for pairwise comparisons of y_i with y_j, y_j with y_k and y_i with y_k. At this time we also know that these responses do not contradict each other. We assign each of these responses to P_{DM} or I_{DM} accordingly and test the transitive closure of the obtained relation as in the initial interview with the DM. Subsequently the system carries out further formation of the binary relations on L. Information on pairs of vectors from L for which the relation has not been defined is inferred from the previous responses of the DM. This is followed by again testing for transitive closure.

This approach guarantees that previous DM responses do not contradict the newly built relation (because we only use responses for those pairs of vectors for which previous responses have not predefined some relation). As a result we obtain a new transitive relation on the set L in which the necessary changes have been made but all previous responses not contradictory to the new ones are maintained unchanged. After that the condition of 'double test' for each pairwise comparison is carried out for this new information. The proposed approach makes it possible to form an effective procedure for an interview with a DM to build the required relation, since the redundancy of the obtained information is limited to a reasonable minimal condition necessary to test DM responses.

2.4. Construction of the joint ordinal scale

As a result of an interview with a DM and transforming his or her responses to a non-contradictory set, the transitive and reflexive relation $R = P_{DM} \cup I_{DM}$ of a linear quasi-order on the set L is built. Information from comparison of vector pairs near the reference point may be used for construction of the joint ordinal scale (Larichev *et al.*, 1974; Ozernoy and Gaft, 1978). The joint ordinal scale (JOS) in these works was considered to be the ranking of vectors from L. Since the binary relation built on the set L is a connected one, the linear quasi-order is defined (in some cases this relation may be a linear order). Therefore we are able to assign a number to each vector from L representing that vector's place in the rank order.

If we recall that vectors near the reference point differ from that reference point in only one component, the rest of such vectors consisting of ones, we can consider that unique number to be the initial ranking of that vector. That is why this ranking was called a joint ordinal scale (JOS). An example of such a JOS is presented in Figure 2. A rule for comparison of any vectors on Y may be formulated from the JOS. The correctness of the rule can be proven for the case of pairwise preferential independence of all criteria (Gnedenko *et al.*, 1986, Larichev and Moshkovich, 1991). Maintaining the formal adequacy of the statement, let us formulate it in a more precise way.

Let $L' = L \cup (1,1,\ldots,1)$, i.e. L' is formed from L by combining it with a vector containing all the best values (1s). It is reasonable to enter this vector into the JOS as the most preferred vector. Let us complement the relation R by relations which reflect the preference of the vector $(1,1,\ldots,1)$ to all other vectors from L and its equality to itself. Then the statement may be formulated in the following way.

Statement 1

If each pair of criteria from K does not depend preferentially on other criteria, then vector $y_i = (y_{i1}, y_{i2}, \ldots, y_{iQ}) \in Y$ is not less preferable for a DM than vector $y_j = (y_{j1}, y_{j2}, \ldots, y_{jQ}) \in Y$ (this means that $(y_i, y_j) \in R$) if for all criteria $s \in K$ there exists a criterion $t(s) \in K$ such that $(1,1,\ldots,1,y_{is},1,\ldots 1) R (1,1,\ldots,1,y_{jt},1,\ldots,1)$. At the same time the criteria $s,q \in K$ are such that if $s \neq q$, then $t(s) \neq t(q)$.

(Proof of Statement 1 is given in Appendix II.)

Thus, to effectively use the information obtained from a DM, it is necessary to have preferential independence of all pairs of criteria (Keeney, 1974).

Definition 3

Criteria s and t of the set K are preferentially independent from other criteria of this set if preference between vectors with equal components upon all criteria but s and t does not depend on the values of these equal components.

RANK	estimate	vector
1	estimate 2 of criterion 2	12111
2	estimate 2 of criterion 3 estimate 2 of criterion 5	11211 11112
3	estimate 3 of criterion 3	11311
4	estimate 2 of criterion 1	21111
5	estimate 2 of criterion 4	11121
6	estimate 3 of criterion 2 estimate 3 of criterion 4	13111 11131
7	estimate 3 of criterion 1	31111
8	estimate 4 of criterion 3	11411
9	estimate 3 of criterion 5	11113

Figure 2. Joint ordinal scale

In practical problems it is necessary to check if this axiom is not violated in DM preferences. The problem of checking this axiom (as well as checking many other axioms of multiattribute utility theory) has no simple solution. In reality, the necessity to use this axiom results from the desire to construct an effective decision rule on the basis of a relatively small amount of rather simple information about DM preferences (the effectiveness of the decision rule means that it is possible to guarantee a rather high level of compatability for real alternatives). On the other hand, a full-scale check of DM preferences implies the need for the DM to carry out a large number of pairwise comparisons of vectors (in some tasks an enormously large number). Thus the point is to make sufficient representative checks of DM preferences to satisfy the axiom's conditions. In Gnedenko *et al.* (1986) a special procedure for checking the axiom is proposed.

It is based on comparison by the DM of additional pairs of vectors from Y, analogous to those from L compared previously by the DM.

Definition 4

The pair of vectors $(y_i', y_j') \in Y \times Y$ is called analogous to the pair of vectors $(y_i, y_j) \in L \times L$ if

$$y_i = (1,1,\ldots,y_{is},1,\ldots,1), \qquad y_j = (1,1,\ldots,y_{jt},1,\ldots,1)$$
$$y_i' = (n_1,n_2,\ldots,n_{s-1},y_{is},n_{s+1},\ldots,n_{t-1},1,n_{t+1},\ldots,n_Q)$$
$$y_j' = (n_1,n_2,\ldots,n_{s-1},1,n_{s+1},\ldots,n_{t-1},y_{jt},n_{t+1},\ldots,n_Q)$$

The two pairs of alternatives differ only in components upon criteria s and t. Thus pairs differ from one another only in values of equal components. Therefore, if criteria s and t are preferentially independent, the preference in these two pairs of vectors (y_i, y_j) and (y_i', y_j') has to be the same.

2.5. Implementation of the joint ordinal scale for comparison of real alternatives

As has been stated above, the JOS may be used for pairwise comparison of real alternatives with the help of a decision rule, based on component comparison of ranks of values upon criteria, determined by the JOS. This comparison may be carried out according to the following procedure.

Consider two vectors $y_i = (y_{i1}, y_{i2}, \ldots, y_{iQ})$ and $y_j = (y_{j1}, y_{j2}, \ldots, y_{jQ})$. Assign to each component of y_{iq} the number r_{iq}, equal to the rank of the vector from L in the JOS, which has the value y_{iq} upon criterion q and the best values upon other criteria. In the resulting vector $r_i = (r_{i1}, r_{i2}, \ldots, r_{iq})$ renumber components in an ascending sequence. Therefore $r_{i(1)} \leqslant r_{i(2)} \leqslant \ldots \leqslant r_{i(q)}$, where $r_{i(q)}$ is the component of r_i which has the qth number.

Similarly, we shall get vector r_j corresponding to vector y_j. Then, for y_i to be not less preferable than y_j, it will be sufficient that $\forall q = 1,2,\ldots,Q, r_{i(q)} \leqslant r_{j(q)}$. (The proof is evident. Actually, as has been stated above, for y_i to be not less preferable than y_j, it is enough if for each component of vector y_i there is not a more preferable component of vector y_j. The ranks introduced reflect the preferability of one's values upon the other as expressed in the JOS. Therefore the smaller rank reflects the more preferable value. As a result, if we order components of vectors y_i and y_j in accordance with the ranks of the JOS, then the first component will correspond to the most preferable value of this vector. If the most attractive component of vector y_j has smaller rank, it indicates at once that it is impossible for vector y_i to be more preferable than vector y_j and so on.)

This gives us a simple procedure for comparison of real alternatives and allows us to give the decision maker a simple explanation of the result (see Figure 3 for an example).

2.6 Rank order of alternatives

Assume a set of real alternatives estimated upon the set of criteria. On the basis of the above rule for pairwise comparison of any vectors from Y, the matrix of pairwise comparisons of real alternatives can be constructed. The quasi-order built on the set of real alternatives on the basis of this rule may be unconnected (not all pairs of alternatives have to be compared). Therefore the problem of ranking the real alternatives arises because different principles of ordering may in general lead to different results.

To solve this problem, different principles of alternative ranking were applied (Larichev and Moshkovich, 1991) on the basis of the matrix of pairwise comparisons according to the DM's choice. Four main principles were used:

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ALTERNATIVE  f010 (vector   12121)
             IS MORE PREFERABLE THAN
ALTERNATIVE  f008 (vector   22211)
      because as a result of the interview it is stated that:

estimate 1 upon criterion 1 (alt. f010) IS MORE PREFERABLE THAN
estimate 2 upon criterion 1 (alt. f008);
estimate 2 upon criterion 2 (alt. f010) IS EQUAL TO
estimate 2 upon criterion 2 (alt. f008);
estimate 1 upon criterion 3 (alt. f010) IS EQUAL TO
estimate 1 upon criterion 4 (alt. f008);
estimate 2 upon criterion 4 (alt. f010) IS MORE PREFERABLE THAN
estimate 2 upon criterion 3 (alt. f008);
estimate 1 upon criterion 5 (alt. f010) IS EQUAL TO
estimate 1 upon criterion 5 (alt. f008);
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Figure 3. Visualization of explanations in the system of alternatives' comparison on the basis of the JOS

(1) the sequential selection of non-dominated alternatives;
(2) the sequential selection of non-dominating alternatives;
(3) the sequential selection of alternatives dominating the maximum number of other alternatives;
(4) the sequential selection of alternatives dominated by the minimal number of other alternatives.

The ranking of initial alternatives upon all principles with computation of the average rank for each alternative may be carried out. (This information may be used for sensitivity analysis of the alternative's place in different orderings.)

The system ZAPROS, implementing this method (for more details see Larichev and Moshkovich (1991)), was used in experiments. The ZAPROS system aids in construction of a quasi-order on the set of alternatives on the basis of a decision maker's preferences. It uses qualitative DM expression of preferences and implements a logical decision rule over available alternatives. Ordinal scales are used for criteria. These ordinal scales are expressed in verbal form. The source of information about the DM's preference system is an interview. Responses are checked by the system to detect possible errors in his or her responses. The DM can correct such errors in dialogue with the system. The system retains information which can be used to explain obtained results.

3. EXPERIMENTAL DESIGN

The ZAPROS system was compared with two well-known existing systems, the preference cone and AHP methods, in order to establish the appropriateness of its use for standard multiattribute problems. The preference cone method (Koksalan *et al.*, 1984; Korhonen *et al.*, 1984; Ramesh *et al.*, 1988) provides a means to apply ordinal preference information based upon pairwise comparisons of vectors of attributes to a large database of alternatives. Preference cones aid in selecting a preferred alternative but will not support a ranking task. Vectors measuring attainment on all criteria for two alternatives are presented to the decision maker, who selects the most preferred of the two alternatives. A preference cone reflecting this choice, based upon the implied trade-off in criteria attainments, is tested on all remaining alternatives. The trade-off between the rejected alternative and the tested alternative is checked through a small linear programming model to see if the tested alternative could possibly be selected by a consistent decision maker. The method can speed the decision process by eliminating rejected alternatives from DM consideration. See Koksalan (1989) for discussion. Koksalan *et al.* (1988) demonstrated that when ordinal preference information is used, the cardinal preference cone approach does not guarantee the selection of the most preferred solution. That paper presented some heuristics to improve the probability that the most preferred solution would be selected. In the study that we are presenting, the preference cone method presented in Korhonen *et al.* (1984) was used. Here, because the criteria are ordinal, that method is heuristic.

The analytic hierarchy process (Saaty, 1977) is a technique which uses DM ratio pairwise comparison of hierarchy elements and alternative performances to obtain a cardinal value for alternatives which can be used as the basis for selection or ranking. The first step is for the decision maker to develop a list of criteria. Then the decision maker places these criteria in a hierarchy, grouping related criteria together, using the hierarchy to cluster subcriteria below criteria. Once the hierarchy is constructed, the relative importance of each branch at a hierarchical node is determined by DM pairwise comparison of the ratio of relative importance between each of the node's branches. At each node i, all branches j being compared have implied ratio weights w_{ij}, with $\Sigma_j w_{ij} = 1$. Subelements of the hierarchy further subdivide the weights w_{ij}. The bottom level of the hierarchy continues this process, with the j elements consisting of the relative performance of each alternative on each subcriterion. Then scores for each alternative are obtained by summing the weighted relative attainment of alternative m over the hierarchy. The eigenvector method is used to rectify DM inconsistencies, providing a check to flag the decision maker if pairwise comparisons are too inconsistent.

3.1. ZAPROS and preference cones

First we compared ZAPROS with the preference cone method. Both methods are able to work with ordinal information. Both require pairwise comparison of multiattribute alternatives, though in different manners. With ZAPROS, subjects compare attribute features differing in estimates upon only two criteria and there is some feedback for their judgments since the system is able to detect contradictions and correct them. With preference cones, subjects compare real alternatives (which usually differ upon a large number of criteria) and there is no possibility to change responses.

We developed a sample of 30 proposals of jobs for students to select from. Each job had a verbal short description. Since ZAPROS works with ordinal verbal scales, we elaborated five attributes—Job Type, Location, Salary, Training and Promotion—to characterize each job and formed three- or four-point scales for them (see Appendix I). Each student was asked to categorize each job according to these attributes. Subjects (28 senior undergraduate students, all of whom were at the job-seeking stage) had the assignment to work with ZAPROS and the system for preference cones using the same sample of jobs. ZAPROS was modified slightly for the experiment in order to make it easier and quicker. This meant that the analysis for attribute preferential independence was not carried out; a check was inserted which did not allow subjects to change the order of values on attribute scales during the interview and the alternatives were rank ordered on the basis of the principle of non-dominated alternatives.

After working with the systems, subjects had to respond to a questionnaire for each system (see Appendix III), which involved answering six questions concerning the ease and practicality of working with the systems and the quality of the result. To have some basis for comparison of the results after working with the systems, subjects were also asked to rank order five to eight of the more preferable jobs according to ZAPROS and preference cones.

3.2. ZAPROS and AHP

For the second test we compared ZAPROS with AHP. ZAPROS is designed for rank ordering of alternatives, so it is attractive to compare the results with some technique which also supports rank ordering of alternatives. In addition, it was of interest to compare results obtained with attribute/criterion analysis but not using ordinal scales (as in ZAPROS). AHP uses ratio scales. The students were first asked to work with AHP. Twelve jobs from the database of 30 used for preference cones were included. Their descriptions included nine criteria which were presented in a qualitative form (with the exception of starting salary). Each student was asked to select five to seven alternatives that were the most attractive to the subjects from this set. Students conducted an AHP analysis on these five to seven alternatives, including development of their individual hierarchies of objectives and assessments of alternative relative performance on each objective. Students were then asked to rank order these five to seven selected alternatives (individual ranking based upon subject post-analysis opinion). Note that this individual ranking is not considered necessarily accurate, but provides some evidence of true preference. The joint ordinal scale from the prior ZAPROS analysis was used to rank order the same five to seven alternatives. Subjects responded to the questionnaire in Appendix III for each method.

4. EXPERIMENTAL RESULTS

4.1. ZAPROS versus preference cones

In Table I the average data on subject responses to the questions given in Appendix III for each system are presented.

Student evaluations of both systems are rather high upon all criteria addressed in the questionnaire. (The questionnaire results were intended to identify subjective opinions of the usability of ZAPROS, given subject exposure to alternative systems. Therefore the results are not presented as a basis for generalizable significance.) On the whole, there is little difference in estimates between the two systems. Nevertheless, we may note the following. Responses to the first question indicate that the students found preference cones easier to work with. This is understandable, because the preference cone method requires fewer iterations, takes very little time and the result is a direct consequence of the last student selection. These features also explain the slight preference for preference cones over ZAPROS in question 3 (understandability of the result) and question 4 (how quickly the result was obtained). It is more difficult to explain the results for questions 2, 5 and 6. There is a slight preference for ZAPROS

Table I. Questionnaire results—comparing ZAPROS and preference cones

		Mean response		Preferred system	
	Scale	ZAPROS	Preference cone	ZAPROS	Preference cone
Question 1. Convenience	1–5	2.1	1.7	4	11
Question 2. Satisfaction	1–5	2.3	2.5	10	5
Question 3. Understandable	1–3	1.6	1.6	6	5
Question 4. Quick	1–5	1.9	1.8	6	8
Question 5. Useful	1–3	2.1	2.0	2	5
Question 6. Would use	1–3	2.0	1.9	2	6

Regarding scale, 1 indicates most preferred.
When the number of preferences does not add up to the number of respondents (28), no preference between the two systems was expressed by some of the respondents.

in the responses to question 2 (satisfaction with the result), which can be explained by the feedback presented in ZAPROS. This feedback in ZAPROS avoids the situation, possible in preference cones, where erroneous comparison of two alternatives leads to the loss of the subject's best solution. However, this explanation is somewhat contradicted by the slight preference for preference cones in the responses to questions 5 and 6 (concerning the usefulness of the system in real choice).

The conclusion may be made that the students prefer to work with real alternatives (since they more commonly use such a mode of information processing), although they do not realize that they may often make mistakes, some rather obvious. (In psychological research, Slovic *et al.* (1977) noted that people are often overconfident about their judgment accuracy.) However, on the whole, both systems are good enough for all subjects and differences are slight.

We also compared the best alternatives obtained through both systems with the individual subject ranking of alternatives. To characterize the process, Table II gives data on the number of comparisons carried out by the subjects while working with both systems and the number of contradictory responses which led to a change in previous judgments in the ZAPROS analysis.

Table II. First-choice selections by technique Z, ZAPROS; PC, preference cone; IR, individual ranking

	Best alternative			Number of comparisons		Number of changes for ZAPROS
	Z	PC	IR	Z	PC	
1	16	16	16	16	8	2
2	1 = 4	4	1	15	11	0
3	17	17	17	22	11	0
4	4 = 16 = 24	5	1	12	8	0
5	1	1	1	22	12	1
6	16	16	16	20	9	0
7	28	28	7	16	11	1
8	16	16	16	23	9	0
9	8	1	8	16	6	0
10	15	15	15	9	9	2
11	16	5	16	8	7	2
12	16	16	16	10	9	1
13	17	17	17	16	9	0
14	1	16	16 = 1	15	11	1
15	1 = 15	1	16	17	10	0
16	16	1	1	20	14	0
17	1	1	1	13	8	0
18	1 = 16 = 28	28	1	17	8	6
19	1 = 15	15	15	21	8	0
20	5	5	13	24	10	0
21	1	27	17	11	6	1
22	1	1	1	12	11	1
23	16	4	16	14	7	0
24	28	28	15	21	11	0
25	1 = 15	28	1	19	10	0
26	28	28	16	17	11	0
27	16 = 15	15	17	16	9	1
28	16	4	16	25	9	0

Number of times the result was the same in Z and PC, 18.
Number of times the result was the same in Z and IR, 19.
Number of times the result was the same in PC and IR, 13.

We conclude that both approaches give the 'right' answer in the majority of cases—but what are the reasons for the differences? To our mind the reason is possible inconsistencies in the subject responses, but not the ideas of the methods. Both methods (and also individual ranking) provide a sound basis for the selection of the best alternative. Also, as we can see in the majority of cases, both approaches lead to one and the same result. However, we can also see that neither system guarantees the 'correct' result.

Let us analyse more thoroughly the differences in judgments that led to different results for ZAPROS and preference cones. Nine subjects obtained different rank ordering of alternatives across the two methods. What is the reason for this? As far as ZAPROS is concerned, we may say that possibly the algorithm for pairwise comparison of real alternatives on the basis of a joint ordinal scale is not appropriate. (The joint ordinal scale can be considered to be accurate, since it was checked for consistency in the interview.) If this explanation is accepted, we must say that people are good at comparison of real alternatives and the correct answer is obtained through preference cones.

However, we have additional evidence of student ranking of alternatives. The subjects rank ordered alternatives. Let us consider the pairwise comparison of alternatives obtained as a result of this ranking and the pairwise comparisons carried out through preference cones. We have 14 cases in which the result of the preference cone analysis does not coincide with the individual ranking. These cases indicate instability in preferences for these subjects. Thus we will analyse those instances where there were differences in results by method. There were rather a large number of subjects with contradictory conclusions. In the case of preference cones and individual ranking we were not able to check such inconsistencies and to aid subjects in analysing the situation. We may say that in most cases subjects reversed their preferences when alternatives were very close for them in quality (almost equal or equal). However, if we analyse more data for preference cones on pairwise comparisons, we can find cases where subjects made rather rough mistakes in comparisons (preferring inferior alternatives). Usually in these cases the subjects were not satisfied with the result (according to the questionnaire), since this mistake did not allow them to identify the best alternative.

Therefore we can conclude that in difficult cases (when many attributes are present and alternatives are close in quality) people tend to make mistakes. ZAPROS provides a necessary element for the preference elicitation process by including a consistency check. Because ZAPROS provides the same result as preference cones or individual ranking in less difficult cases, we may conclude that in complicated cases it also guarantees (to a large extent) the correct result.

4.2. ZAPROS versus AHP

Eighteen subjects out of 28 working with ZAPROS and preference cones also worked with AHP. In Table III the average data on subject responses to the questions given in Appendix III for ZAPROS and AHP are presented.

As in the previous case, we can say that both methods were rather highly appreciated by subjects. It is easy to explain why ZAPROS was considered to be quicker in obtaining a result and a bit easier to work with. The students were required to conduct the AHP analysis with only eigenvector calculation support from the computer. This also explains why AHP was more understandable for the subjects. They were given a lecture on this method and the method is well known. Possibly that is why AHP was rated as slightly more useful (question 5). However, it is interesting to note that the result obtained by ZAPROS was more satisfactory to a number of the subjects. On the whole, the differences are insufficient to make any special conclusions. However, both methods seem reasonable to subjects in selection tasks.

Table III. Questionnaire results—comparing ZAPROS and AHP

		Mean response		Preferred system	
	Scale	ZAPROS	AHP	ZAPROS	AHP
Question 1. Convenience	1–5	2.0	2.2	8	5
Question 2. Satisfaction	1–5	2.1	2.3	8	4
Question 3. Understandable	1–3	1.6	1.4	2	4
Question 4. Quick	1–5	1.8	2.6	9	1
Question 5. Useful	1–3	2.0	1.9	0	4
Question 6. Would use	1–3	2.0	2.0	3	2

Regarding scale, 1 indicates most preferred.
When the number of preferences does not add up to the number of respondents (18), no preference between the two systems was expressed by some of the respondents.

Further analysis was concerned with the final ranking of alternatives obtained through ZAPROS, AHP and individual ranking of alternatives. We analysed the coincidence of the first (most preferred alternatives), of the first two (maybe not in the same order) and of the first three in the ranking. The data are presented in Table IV.

As we can see, the data are analogous to the previous case, the proportion of matching the first choice of alternatives being about 60%–70% with any technique. As far as ranking is concerned (dealing with alternatives very similar in quality), we can see that full coincidence in ranking is rare. Let us examine the differences in rankings between ZAPROS and AHP more closely.

Table IV. Mutual alternatives as first, second and third choices in the ranking (Z, ZAPROS; AHP, analytic hierarchy process; IR, individual ranking)

				Coincidence in alternatives								
	Best alternative			Best			Two best			Three best		
	Z	AHP	IR	Z–AHP	Z–IR	AHP–IR	Z–AHP	Z–IR	AHP–IR	Z–AHP	Z–IR	AHP–IR
1	16	16	16	+	+	+	–	+	–	–	–	–
2	1	1	1	+	+	+	+	+	+	+	–	–
3	17	17	17	+	+	+	–	–	+	–	–	+
5	1	1	1	+	+	+	–	+	–	+	+	+
6	16	1	16	–	+	–	+	+	+	–	–	+
7	16	16	15	+	–	–	–	+	–	–	+	–
8	17	1	1	–	–	+	+	–	–	–	–	–
9	1	1	1	+	+	+	+	+	+	+	+	+
10	15	15	15	+	+	+	+	–	–	–	+	+
12	16	1	1	–	–	+	+	+	+	+	+	+
13	17	15	7	–	–	–	–	–	–	–	–	–
15	1	1	1	+	+	+	+	+	–	+	+	+
17	1	15	1	–	+	–	–	+	–	–	+	–
19	15	15	15	+	+	+	+	+	+	+	–	–
20	13	13	13	+	+	+	–	+	–	–	–	–
21	1	1	1	+	+	+	–	–	+	–	–	–
23	16	1	16	–	+	–	–	+	–	–	+	–
28	16	16	7	+	–	–	+	–	–	–	–	–
Total (+)				12	13	12	9	12	7	6	8	7

AHP is based on procedures to assign weights to the criteria and to values of alternatives upon these criteria. The procedures are based on ratio pairwise comparisons of the corresponding elements (criteria and criterion values) and require consistency of these comparisons (in the sense of eigenvector calculation). The procedure detects inconsistencies and stimulates subjects to change some of their estimates. In this sense, as in ZAPROS, we have consistent information which is used to calculate the final aggregate 'values' of alternatives. Thus the differences in relative estimation of alternatives through ZAPROS and AHP must be due to differences in subjects' estimation of alternatives' criterion values and/or the importance of criteria.

Let us now analyse cases in which ZAPROS and AHP selected different best alternatives and try to explain what differences in subjects' judgments led to these results. We have six such cases. For subjects 6, 8 and 12, although the best alternatives were different, the first two best alternatives were the same. This means that these two alternatives were very similar in quality and both systems found this. Their differences in places were probably due to minor differences in the scaling of subjects' responses.

For the other three subjects a detailed analysis of the data was carried out. First of all, the order of criteria was checked, as well as the order of alternatives upon each criterion.

Let us note that while working with AHP, subjects individually formed the set of criteria (and the hierarchy, if necessary) from the descriptions of the jobs. Therefore the criteria were not necessarily the same as in ZAPROS. Nevertheless, almost all of them used most of these criteria at the lowest level of the hierarchy. For all three subjects in question, calculations based upon only the criteria used in the ZAPROS study yielded the same order of alternatives' overall scorings as in the initial AHP study. Thus it was possible to concentrate on just these criteria.

Rank order of criteria for subject 13:

ZAPROS $1>3=2>4=5$
AHP $3>1>2>4$ (no 5)

The rank order is almost the same. (The reversal of criteria 1 and 3 was common, because in AHP almost all subjects have Salary, criterion 3, as the most important one.)

Let us now consider the rank order of alternatives upon criteria scales. Using ZAPROS, alternative 17 was selected, and using AHP, alternative 15. Analysing the data, we found that in the AHP analysis alternative 15 was preferred to alternative 17 upon criterion 1 (Job Type). In the ZAPROS analysis alternative 17 was preferred to alternative 15 upon the same criterion. Let us note that in assessing estimates to alternatives in ZAPROS, subjects used a very simple verbal scale with three possible gradations (see Appendix I). Thus the difference between alternatives 15 and 17 upon this criterion would be rather clear to merit a misclassification (one for alternative 17 and two for alternative 15). This is a reason for the difference in the result. We may assume that the categorization carried out in ZAPROS was not right (since these judgments were not tested).

Rank order of criteria for subject 17:

ZAPROS $2>1>3>5>4$
AHP $2>1>3>4>5$

In this case the order of criteria is almost identical for the two systems. We analysed the criteria values for alternatives 1 and 15 obtained in ZAPROS and AHP and found that they were in the same order upon all criteria. Alternative 15 was better than alternative 1 only upon criterion 1 (Job Type). However, alternative 15 was distinctly inferior upon criteria 3–5. (The values were almost the same for criterion 2.) Therefore the difference in the selection was caused by the fact that the subject (in his or her joint ordinal scale) marked that the decrease upon the first criterion from the best to the second value is more preferable for him or her than the decrease upon criterion 3 (Salary) from the first to the third value. According to the ZAPROS analysis, alternative 1 (21121) was preferred to alternative 15 (11333). However, in the AHP analysis the difference in Job Type (criterion 1) was estimated to be much higher, as was the criterion 1 weight (which was not confirmed through ZAPROS). Here we may have the effect described in Belton (1986), when the 1-9 scale used in AHP for comparison of criteria is not considered by the subject as a ratio scale and subject response that a value (or criterion) is strongly preferred to another (5 on the AHP scale) does not imply subject intention of five times preference in the relative scores. This is supported by the fact that in individual ranking of alternatives this subject also selected alternative 1 as the best one. We recognize that although we have stated that there is no basis for concluding that the individual ranking is correct in any way.

Rank order of criteria for subject 23:

ZAPROS $3>2>1>4>5$
AHP $3>5>1>4>2$

We can see that the importance of criterion 5 was different in these two systems. Possibly the subject changed his or her mind during the period between working with these two systems. In this case it is clear that the different choice was caused by the differences in the rank order of criteria, since alternative 16 (selected by ZAPROS) was better than alternative 1 (selected by AHP) upon all criteria but Salary (criterion 3), where they were almost equal, and Promotion (criterion 5), upon which alternative 16 was much less preferable than alternative 1. In this case again the subject made the same choice in individual ranking (as in ZAPROS), though this selection was made at the time the subject was working with AHP (not ZAPROS).

5. DISCUSSION

First, each method was initially oriented on different types of decision tasks. Preference cones and AHP were focused on the selection of the best alternative, although AHP may also be used for rank ordering of the whole set of alternatives. Preference cones cannot fully rank alternatives. ZAPROS was developed to construct a quasi-order on a rather large set of alternatives, but in practical cases, as we have seen, it may yield results useful in selection or ranking, although unique solutions are not guaranteed.

Secondly, these methods are oriented on different task dimensionality: ZAPROS and preference cones are useful in tasks where we have a rather large number of alternatives, while AHP is able to comfortably work with only a small number of alternatives (up to seven). While absolute AHP has been proposed for more alternatives, Saaty (1988) emphasizes that the method is expected to be less precise under these conditions. It is useful to note that ZAPROS is able to work with hundreds of alternatives, because it constructs the decision rule (JOS) in the criteria space independently of the set of real alternatives. Preference cones, on the other hand, work with real alternatives. While the cone developed by the preference cone technique should eliminate a larger proportion of alternatives in larger sets, there would still be an expected increase in comparisons required with larger sets. Conversely, the number of criteria influences the number of judgments required in ZAPROS and AHP, while not being as critical for preference cones. That is why, when we have a small number of alternatives estimated upon a large number of criteria, ZAPROS is much less effective than AHP and even preference cones.

Thirdly, all these methods require different types of judgments from a DM. Preference cones ask a DM to compare pairs of real alternatives. It is evident that in many cases, especially when the number of criteria is rather large, this task is rather difficult for a DM and may lead, as we have seen, to contradictory responses. ZAPROS also asks a DM to compare multiattribute alternatives, but they differ in values upon only two criteria. It is known from psychological investigations that such a judgment type is much easier for people, especially since they have an opportunity to say that the two alternatives are equally preferable for them. In AHP simple objects (pairs of criteria and values of two alternatives upon one criterion) are compared using a 1–9 ordinal scale. Such judgments would be easier if this were not meant as a ratio scale. Use of the ratio scale makes the judgment more difficult for subjects according to some studies (e.g. Belton, 1986).

Fourthly, an important characteristic of the methods is their ability to detect possible inconsistencies in a DM's judgments (since people may always make a mistake). This possibility is present in ZAPROS and AHP but absent in preference cones, making it possible to lose the best decision if a DM made a mistake choosing between two presented alternatives.

Fifthly, there may be different cases due to the uniqueness of the task under consideration. Since preference cones and AHP work with real alternatives, usually it is not possible to use the results for choosing the best alternative if the set of alternatives has been changed. Working with AHP, we are able to maintain criteria weights for the different sets of alternatives, but nevertheless we have to work out new value scores for alternatives. On this factor ZAPROS is much more appropriate for tasks where one has to rank order different sets of alternatives estimated upon the same set of criteria, since it creates some sort of knowledge base of a DM's preferences.

Comparative evaluations are recapitulated in Table V.

Table V. Comparison of method characteristics

	Method		
Characteristic	ZAPROS	Preference cone	AHP
I. Type of task			
Selection of best alternative	+ −	+	+
Rank order of alternatives	+ −	−	+
Partial ordering	+	−	+
II. Dimensionality of task			
Large number of alternatives	+	+	−
Small number of alternatives	+ −	+ −	+
Large number of criteria (with one level of hierarchy)	+ −	+	+
III. Uniqueness of task			
Set of alternatives may be changed	+	−	+
Solution unique to alternative set	+ −	+	+
IV. Cognitive effort of DM			
Simple judgments required	+	−	+
Inconsistency check	+	−	+
Small number of DM comparisons	−	+	−

Key: +, relatively superior on this factor; + −, some positive aspects, some negative aspects on this factor; −, relatively inferior on this factor.

6. CONCLUSIONS

ZAPROS, a method which allows construction of a partial ordering on a large set of multiattribute alternatives in accordance with a DM's preferences, has been described. ZAPROS is based on qualitative judgments and logical transition from a decision maker's judgments to a decision rule for comparison of alternatives.

We found no evidence that ZAPROS is less useful in any sense than preference cones or AHP. Each of the three methods has its own peculiarities which may influence technique selection in specific cases. There are arguable advantages for ZAPROS in that it provides a thorough consistency check and is based upon a sound theoretical basis. We consider ZAPROS to have two relative disadvantages compared with preference cones and AHP. First, we have considered a problem where there is a relatively small number of categories on each criterion scale. We contend that people do group performance into such categories. However, if there are important but slight differences in performance on a criterion: ZAPROS treats all alternatives in the same category identically, while cardinal preference cones and AHP can reflect the full cardinal scale. The other disadvantage of ZAPROS is that the number of questions (comparisons) needed from the decision maker increases with an increase in the number of criteria and values on criteria scales. However, this number does not grow with the number of alternatives to be compared (and this parameter influences the number of comparisons to be made in preference cones and AHP as well). Also, we can say that there are some evident advantages in the ZAPROS system which may be useful in practice. First of all, only very simple and understandable information (judgments) from the decision maker (categorization of alternatives upon small, verbal criteria scales; comparison of alternatives differing in values upon only two criteria) is required. This increases our confidence in the validity of DM judgments obtained. In addition, the system includes tools to detect inconsistencies in DM judgments and to help the decision maker in their correction. In addition, the judgments obtained are not modified (rescaled) in any way. Therefore any result may be explained simply and connected to the decision maker's judgments, and as such, modified if the result does not satisfy the decision maker. Of course, ZAPROS does not guarantee the obligatory comparison of any two alternatives (it does not build full order). However, sometimes small differences in the scores of alternatives obtained by numerical rescaling of DM judgments may not really show the real preference of alternatives and are mostly due to the accepted technique of rescaling. In this case it is possibly better not to compare them and their place in the rank order will be due to their comparisons with other alternatives.

We believe that the proposed method (and system) may find successful applications in many practical problems, especially when it is necessary to select a small subset of alternatives out of a large set. While working with the system, DM learning is fostered and the decision maker's knowledge base can be used to deal with other sets of alternatives.

APPENDIX I: CRITERIA AND CATEGORIES FOR JOB CLASSIFICATION

Criterion 1. Type of job position

1. Type of job position is almost ideal.
2. Type of job position is good enough (in field).
3. Type of job position is not appropriate.

Criterion 2. Job location

1. Location of the job is where you want to be.
2. Location of the job is at some distance from where you want.
3. Job is located far away from where you want.

Criterion 3. Salary

1. The salary is rather high.
2. The salary is on the average level.
3. The salary is a bit lower the average level.
4. The salary is rather poor.

Criterion 4. Possibilities for training

1. There are nice possibilities for training.
2. There are normal possibilities for training.
3. There are minimal (almost none) possibilities for training.

Criterion 5. Possibilities for promotion

1. There are good possibilities for promotion.
2. There are moderate possibilities for promotion.
3. There are almost no possibilities for promotion.

APPENDIX II: PROOF OF STATEMENT 1

According to Theorem 3.7 in Keeney (1974), if each pair of criteria does not depend preferentially on other criteria, then all criteria are mutually preferentially independent.

According to Theorem 3.6 in Keeney (1974), in this case there exists an additive value function

$$v(y_i) = \sum_{iq=1,Q} v_q(y_{iq})$$

for criteria of the set K. Let there be two vectors from Y:

$$y_i = (y_{i1}, y_{i2}, \ldots, y_{iQ}), \qquad y_j = (y_{j1}, y_{i2}, \ldots, y_{jQ})$$

Then, according to the initial condition, for all y_{is} there exists a y_{jt} which with all other first values upon the rest of the criteria is not less preferable than this vector. Let the following:

(1) $(y_{i1}, 1, \ldots, 1) R (1, y_{j2}, 1, \ldots, 1)$;
(2) $(1, y_{i2}, 1, \ldots, 1) R (1, 1, y_{j3}, 1, \ldots, 1)$;
$\vdots \quad \vdots$
(Q) $(1, 1, \ldots, 1, y_{iq}) R (y_{j1} 1, \ldots, 1)$.

Expression (1) means that

$$v_1(y_{i1}) + v_2(1) + \ldots + v_Q(1) \geqslant v_1(1) + v_2(y_{j2}) + v_3(1) + \ldots + v_Q(1)$$

and as a result

$$v_1(y_{i1}) + v_2(1) \geqslant v_1(1) + v_2(y_{j2})$$

Expression (2) means that

$$v_1(1) + v_2(y_{i2}) + v_3(1) + \ldots + v_Q(1) \geqslant v_1(1) + v_2(1) + v_3(y_{j3}) + v_4(1) + \ldots + v_Q(1)$$

and as a result

$$v_2(y_{i2}) + v_3(1) \geqslant v_2(1) + v_3(y_{j3})$$

and so on. Let us add the right and left parts of the resulting inequalities, obtaining

$$\begin{aligned} &v_1(y_{i1}) + v_2(1) + v_2(y_{i2}) + v_3(1) + \ldots + v_{Q-1}(y_{iQ-1}) + V_Q(y_{iQ}) + v_1(1) \\ &\geqslant v_1(1) + v_2(1) + v_2(y_{j2}) + v_3(y_{j3}) + \ldots + v_{Q-1}(1) + v_Q(y_Q) + v_Q(1) + v_1(y_{j1}) \end{aligned}$$

Eliminating common elements, we have

$$v_1(y_{i1}) + v_2(y_{i2}) + \ldots + v_Q(y_{iQ}) \geqslant v_1(y_{j1}) + v_2(y_{j2}) + \ldots + v_Q(y_{jQ})$$

Hence $v(y_i) \geqslant v(y_j)$, which is what was required to be proven.

APPENDIX III: QUESTIONNAIRE

Separate pages for ZAPROS, preference cone and AHP.

Question 1. Was it convenient to work with the system?

1. It was easy and comfortable.
2. There were some difficulties while working with the system, but on the whole it was convenient.
3. There were difficulties, but the system may be considered convenient enough.
4. There were essential difficulties. I would say it was not very convenient to work with the system.
5. It was inconvenient to work with the system.

Question 2. To what extent are you satisfied with the result?

1. I fully agree with the result obtained from the system.
2. I almost fully agree with the result obtained from the system.
3. I only to some extent agree with the result obtained from the system.
4. I am doubtful about the result obtained by the system.
5. I am not satisfied with the result obtained from the system.

Question 3. To what extent do you understand the obtained result?

1. The result is easily understandable (according to the information I gave to the system).
2. I am able to see some correspondence between the result and the information I gave to the system.
3. It is difficult for me to understand the result on the basis of the information I gave to the system.

Question 4. How quickly have you obtained the result?

1. The result was obtained quickly.
2. The result was obtained quickly enough.
3. The result was not obtained quickly but in a reasonable time.
4. It took rather a long time to obtain the result.
5. It took a very long time to obtain the result.

Question 5. Was the system useful to you?

1. Work with the system helped to make it clear for me what I want (or prefer).
2. After working with the system I began to understand better what I want (or prefer).
3. Work with the system did not help me in understanding my preferences.

Question 6. Would you like to use the system for real choice?

1. I think that it is better to use this system before making real choice.
2. I admit that this system may be useful in making real choice.
3. I do not think that this system will be useful in making real choice.

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