# Modeling Multiattribute Information Processing Strategies in a Binary Decision Task

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A method is presented for determining the rules underlying a decision maker's binary evaluations of multiattribute stimuli. Two experimental studies showed that the method was successful in determining the rules employed by real decision makers. An algorithm for simplifying the task, so as to detect rules with a minimum of testing, is also presented. These techniques should prove useful in helping decision makers formulate and express their policies.

Judgment of objects characterized by numerous qualities is very difficult. Many investigators have observed that people are inconsistent when making decisions about such complex stimuli (see, e.g., Davis, 1958; Marschak, 1968; Mirkin, 1974; Tversky, 1969). Often a person's judgments of the same object differ from one time to the next. As a result, an algebraic model of the judge, which can be applied consistently, often outperforms (i.e., bootstraps) the judge (Dawes & Corrigan, 1974).

What is the cause of inconsistency? Davis (1958) assumes that one form of inconsistency, i.e., intransitivity, manifests itself primarily when comparing objects that are similar in value. However, later experiments have shown intransitivity even among alternatives of quite dissimilar utility. One extensive analysis of intransitivity and its causes is given by Tversky (1969), who showed how "intransitivity traps" could be constructed for decision makers. When making consecutive pairwise comparisons, the decision makers studied by Tversky repeatedly neglected small changes with respect to the more important attribute in favor of greater changes with respect to a less important attribute. However, in the context of greater changes with respect to the important attribute, the decision maker disregarded previous judgments, thus, intransitivity occurred.

Tversky and others have established the fact that when multiattribute alternatives are evaluated, the decision maker tries to use simplifying techniques such as considering the criteria sequentially (Miller et al., 1960; Simon, 1960; Tversky, 1972). This work demonstrates that the decision maker examines alternatives first on the basis of one criterion attri-

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bute, then on a second, etc., and consecutively excludes alternatives that do not satisfy certain requirements. Such behavior may be due to a desire to decrease the information processing demands of the task. As Simon (1969) observed: "Evidence is overwhelming that the system is basically serial in its operation: that it can process only a few symbols at a time and that symbols being processed must be held in special, limited memory structures whose content can be changed rapidly" (p. 53).

The difficulty of taking into account all properties of compared or estimated alternatives forces people to employ simplifying strategies. However, as Tversky (1969) has indicated, simplifying strategies lead to traps. Note that such traps may also be constructed for other simplifying strategies.

At the same time, there is a practical demand for standard decision models based on the preferences of decision makers. Construction of such models, in our opinion, is possible only on the basis of the results of descriptive studies of what people can and cannot do.

### Reliability Criteria for Obtained Information

When a technique is proposed for obtaining information from an expert, criteria of reliability should be examined. Two criteria, test-retest consistency and intransitivity, were mentioned above.

Most generally recognized is the transitivity criterion (Slovic & Tversky, 1974). As a rule, people want to be transitive. Consistency is another natural criterion. When the situation requiring evaluation or comparison of alternatives is the same on two or more separate occasions, decisions should be consistent.

To our mind, it is reasonable to introduce a third criterion, namely the consistent expression of complex strategies involving various combinations of criteria. Under certain preference elicitation techniques, people may resort to simplifying strategies such as successive elimination of alternatives on the basis of various criteria. Various preference elicitation or detection techniques may actually induce consistent and systematic policies. In some cases, however, these policies are forced to be simplified because the decision makers meet with difficulties in expressing their preferences. Understandably, preference detection techniques where subjects use simplified strategies cannot be regarded as trustworthy.

Application of the above three criteria to modeling experiments and actual problems is based on certain concepts about the decision makers' capabilities. We have in mind experienced decision makers who well understand the actual decision-making situation. The decision maker's ability to express a policy depends not only on the specific personal preferences, but also on the difficulty of the problem being faced (e.g., comparison of multidimensional alternatives, determination of ranks, assignment of criterion weights, etc.).

### METHOD

In multicriteria decision-making situations, the rules being applied by the decision maker can sometimes be inferred from the responses to carefully designed questions (Larichev, 1975).

Techniques for inferring decision rules should be verified through specially constructed modeling experiments. If these experiments find that the above three criteria are satisfied (i.e., people make nonrandom judgments, a high percentage of their judgments are consistent and noncontradictory, and they use complex strategies), then the preference detection technique under study may be useful. If the results are doubtful, even for only one of these three criteria, careful analysis of the reasons for this failure is necessary, followed by new hypotheses and new experiments.

If the results of the modeling experiments are satisfactory, they should be supplemented by further study in real decision-making situations. In real situations, of course, the motivation of decision makers is higher, and their understanding of their strategy is better as compared with that of subjects in an experimental situation. At the same time, positive results of hypothesis verification in modeling experiments give promise that follow-up work with real decision makers will be successful. Without this experimental pretesting, there is danger of asking questions of real decision makers that are too complicated, and of obtaining answers that contain numerous mistakes or are obtained through simplification of their true preferences.

## Binary Decision Making

Larichev *et al.* (1978) studied judgment situations in which the attributes of each object were defined in terms of a relatively small number (e.g., two or three) of verbal labels. In a number of practical problems, one can assume that the decision to be made has two levels of quality, good or bad (sometimes this binary scale can be obtained by merging judgments made on a scale with many levels).

When evaluating alternatives having various levels of a hierarchical multiattribute structure, the decision maker has to be able to trace the determinants of the decision down to the effects of specific attributes (Larichev *et al.*, 1977). A simple way to achieve this is to use at each level of the multiattribute system only two levels of evaluation, good and bad.

This has led us to study problems involving N attributes, each of which has only binary levels, and two classes of final decision. We thus have assumed that direct evaluation of these alternatives is not beyond the

decision maker's powers, even when a large number of attributes are involved.

The following hypothesis was formulated and tested: If the number of attributes is at most seven, and these attributes are binary, and if there are two classes of final decisions, it is possible to obtain reliable information that will permit a decision maker's preference strategies to be inferred.

#### The Experimental Task

Two psychometric experiments were carried out to verify the above hypothesis. The judgment task was the evaluation of various versions of future urban transport systems. Seven attributes, each with two scale levels, were used to characterize these systems (see Table 1).

The subjects were given all possible combinations of estimates with respect to these criteria (126 altogether, excluding the best and the worst alternatives), and they were asked to classify them as either X or Y according to special instructions:

Class X—you would be willing to use the transport vehicle described by these attributes.

#### TABLE 1

### ATTRIBUTES USED TO DESCRIBE TRANSPORT SYSTEMS

Attribute A. Transport speed

1. Transport speed permits you to reach any place (within the city) in 30 minutes at most.

2. Transport speed is similar to the current system.

Attribute B. Reliability of the means of conveyance

- 1. Intervals are such that practically no time is spent waiting.
- 2. Waiting time at stops is up to 10 minutes.

Attribute C. "Door-to-door" delivery

- 1. Municipal transport operates practically in the "door-to-door" mode.
- 2. Present-day time to reach the nearest transport stop is expected to remain unchanged.

Attribute D. Journey comfort

- 1. Each passenger is given a separate comfortable place in the vehicle.
- 2. The passenger may sometimes have to stand.

Attribute E. Journey costs

- 1. Monthly transport costs do not exceed 30% of present-day costs.
- 2. Transport costs correspond to the present-day level of 6 to 7 roubles per month.

Attribute F. Safety of journey in municipal transport

- 1. Road accidents are practically non-existent.
- 2. Number of road accidents corresponds to the present-day level.

Attribute G. Influence on the environment

- 1. Practically no environmental pollution.
- 2. Pollution of environment is at the present-day level.

Class Y—you would not use the transport vehicle described by these attributes.

In doing so, subjects were told that the highest level of quality with respect to all the attributes was unattainable in practice, and they were asked to draw a "boundary line" between the classes using some reasonable principle of compromise.

In the first experiment, seven graduates of the Moscow Automobile and Road Institute served as subjects. The stimuli and instructions were discussed in detail with them, thus insuring unambiguous understanding of concepts used.

Sets of six stimuli were arranged on cards in order of decreasing quality (i.e., all the attributes of each stimulus should not be worse, and one should be better than those of the stimulus that followed). For example:

$$\begin{array}{l} A_2B_1C_1D_1E_1F_1G_1\\ A_2B_2C_1D_1E_1F_1G_1\\ A_2B_2C_2D_1E_1F_1G_1\\ A_2B_2C_2D_2E_1F_1G_1\\ A_2B_2C_2D_2E_2F_1G_1\\ A_2B_2C_2D_2E_2F_2G_1 \end{array}$$

Each card contained five such sets of stimuli.

The following order was imposed for consideration of stimulus elements:

- 1. Evaluate the stimulus in row three of the set on the card.
- 2. If this stimulus belongs to Class X, the next one below is considered, and so on until a stimulus belonging to Class Y occurs or the set is exhausted.
- 3. If the third stimulus belongs to Class Y, the stimulus in the row above it is considered, and so on until a stimulus belonging to Class X is found or the set is exhausted.

Having considered the six alternatives of a given set, the subject passed to another one (seven cards having five sets of six stimuli each were presented altogether). Design of these stimulus sets took advantage of the dominance relations between rows. If the stimulus under consideration was classified as belonging to Class X, the stimulus above it should also belong to this class (due to the dominance relation), and there is no need to estimate it. Similarly, if the stimulus under consideration is classified as belonging to Class Y, all the stimuli below it should belong to the same class.

With such an inquiry procedure, membership of each of the 126 stimuli is determined several times, either directly or through the dominance principle.

In the second experiment, where managerial personnel of trucking

agencies were the subjects, the problem was formulated in broader terms. In addition to the above method, subjects were asked to evaluate separately each of the 126 combinations (arranged in random order on a rating sheet).

In the course of the second experiment, the 24 subjects were assigned to two groups of 12. One group first judged the sets of ordered stimuli (on cards) and then judged all 128 stimuli in random order (on sheets of paper). The second group did these tasks in reverse order.

### RESULTS

### Consistency

We assumed that if a subject classified a given stimulus as X one time and as Y another, one of the answers was mistaken (i.e., there was a contradiction).

To calculate the number of contradictions in subjects' answers, information obtained through the experiment was represented in the following manner. A decision table (see Fig. 1) was constructed where each square represents certain combinations of estimates with respect to the seven criteria and the field of squares covers the entire set (128) of stimulus combinations. Experimental data obtained from each subject (i.e., evaluations given by the subject to each stimulus alternative) were entered into the table. In doing so, alternatives classified as X were colored red, and those classified as Y were colored black.

Such a visual representation of information enabled easy detection of contradictory answers. Next, the minimal number of answers was determined which one had to change in order to obtain a completely consistent picture of subject preferences. The number of such changes of "red" squares for "black" ones and/or "black" squares for "red" ones was taken as the number of contradictions made by the subject.

In Experiment 1, analysis of contradictory judgments resulted in the data shown in Table 2. The number of contradictions of the majority of subjects ranges between two and four, i.e., is 1 to 3% of the total number of stimuli that were evaluated. Similar data were obtained for both groups of the second experiment. The mean values are shown in Table 3.

The data in Tables 2 and 3 were compared to the number of contradictions that would be expected under the assumption that subjects' decisions were made randomly. The hypothesis that the observed number of contradictions resulted from a random strategy was rejected at a high level of statistical significance. Thus, we conclude that the small number of contradictions observed here is due to the fact that subjects here used specific and consistent evaluation strategies.

$\begin{array}{c c} \mathbf{A}_1\mathbf{B}_1 & \mathbf{A}_2\mathbf{B}_1 \\ \hline \mathbf{A}_1\mathbf{B}_2 & \mathbf{A}_2\mathbf{B}_2 \end{array}$		$\frac{   }{   }_{c_2 p_1}$	$\begin{array}{c c} \square \\ \square \\ \square \\ c_1 \\ p_1 \end{array}$	$\begin{array}{c c} & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ &$	$\begin{array}{c c} F_2 \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c c} \hline \\ \hline \\ \hline \\ \hline \\ c_2^{n_1} \end{array}$	$\begin{array}{c c} \square & \square \\ \square & \square \\ \hline \square & \square \\ c_1 p_1 \end{array}$	
								$\begin{array}{c c} \square & \square \\ \hline \square & \square \\ \hline \blacksquare & \blacksquare \\ c_2 D_2 \\ 2 \end{array}$
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							$\frac{\boxed{2}}{\mathbb{2}} \boxed{2}$	$\frac{\boxed{2}}{\underset{2}{\boxtimes}}$

FIG. 1. Method used for displaying responses to detect inconsistencies and decision rules.

## Types of Decision Rules

In further analysis of the experimental results, we attempted to classify decision rules into three categories:

1. Simple decision rules, or "pure cutoff" rules, where the subject classifies a given stimulus as Y if there is at least one attribute that takes level two (the inferior level) (such a strategy undoubtedly simplifies the subject's task);

2. complex decision rules where the subject takes into consideration combinations of estimates with respect to several criteria rather than single criteria;

NUMBER OF CONTRADICTIONS IN EXPERIMENT I											
				Subject	number						
	1	2	3	4	5	6	7	8			
Total number of contradictions	2	2	4	3	7	2	1	4			

TABLE 2

NUMBER O	F CONTRADICTIONS IN EXPER	IMENT II
Group	Mean number of	contradictions
I	Sheets (1st inquiry) 5	Cards (2nd inquiry) 2
II	Sheets (2nd inquiry) 6	Cards (1st inquiry) 3

 TABLE 3

 Number of Contradictions in Experiment II

3. mixed decision rules where estimation is performed both by separate criteria ("pure cutoffs") and by criteria combinations.

Rules 1, 2, and 3 are referred to in the literature as conjunctive, compensatory, and mixed conjunctive-compensatory (Payne, 1976).

To identify the decision strategies used by subjects, contradictions in their decision sets were removed according to an algorithm based on minimizing the number of changes in the answers of subjects. This algorithm occasionally permits two ways of changing a subject's answers. In such cases, when no logical considerations dictated which square color was better to change, the choice of the square was made randomly.

After removal of contradictions in decision sets, the purified sets may be employed to readily identify the decision strategies used by each subject. In fact, the decision strategy may be represented by stimuli (combinations of estimates) of the boundary between classes X and Y. These combinations are such that all stimuli dominating them belong to Class X, and these combinations together with all stimuli dominated by them belong to Class Y. These combinations of estimates can be easily found in purified decision sets and the type of these stimuli (i.e., the number of lower estimates in each stimulus) characterizes the category of decision strategy.

For example. Fig. 2 illustrates the data matrix which characterizes simple decision rules with pure cutoffs based on attributes F and G. Figure 3 illustrates the table characterizing a mixed decision rule ("pure cutoff") on attributes F and G and more complex decision rules for other stimulus evaluations. Figure 1 represents the case of a complex decision rule (without a "pure cutoff" on any attribute).

This sort of analysis revealed that the subjects used all kinds of decision rules; simple rules did not predominate. In the first experiment, four of the eight subjects used simple rules, and the rest used complex ones. All the subjects used at least three attributes for discriminating among the stimuli.

In the second experiment, of 12 subjects who first worked with cards, only 4 used simple decision rules, 5 used complex rules, and 3 used mixed  $\begin{array}{c|c} \begin{array}{c|c} A_1 B_1 & A_2 B_1 \\ \hline & A_1 B_2 & A_2 B_2 \end{array}$ 

 $\begin{array}{c|c} \begin{array}{c|c} A_1 B_1 & A_2 B_1 \\ \hline & A_1 B_2 & A_2 B_2 \end{array}$ 

			$\begin{array}{c c} & & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\$	$\begin{array}{c c} F_2 & & \\ \hline & & \\ & \\$	$\frac{\boxed{\square}}{\boxed{\square}} \frac{\boxed{\square}}{\boxed{\square}}_{c_2^{D_1}}$	$\frac{\boxed{\square}}{\boxed{\square}} \frac{\boxed{\square}}{\boxed{\square}}$	$\frac{\boxed{\square}}{\underset{c_2^{\mathbf{D}_1}}{\boxdot}}$
				$\begin{array}{c c} & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ &$	$\begin{array}{c c} \hline \square & \square \\ \hline \square & \square \\ \hline \square & \square \\ c_2 p_2 \\ 1 \end{array}$		$\begin{array}{c c} & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ &$
$\overbrace{C_1 p_1}^{G_2} \boxed{\square}$	$\frac{\boxed{\square}}{\boxed{\square}} \boxed{\boxed{\square}}_{c_2^{p_1}}$	$\frac{\boxed{\square}}{\boxed{\square}} \boxed{\boxed{\square}}_{c_1 p_1}$	$\frac{\boxed{\square}}{\underset{c_2^{D_1}}{\boxdot}}$	$\begin{array}{c c} & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ &$	$\frac{\boxed{2}}{c_2^{\mathfrak{p}_1}}$	$\frac{\boxed{\square}}{\underset{c_1 p_1}{\square}}$	$\begin{array}{c c} & & & \\ \hline & & \\ \hline & & \\ \hline & & \\ c_2 p_1 \end{array}$
$\begin{array}{c c} & & & \\ \hline & & & \\ & &$						$\begin{array}{c c} & & & \\ \hline & & & \\ & &$	

FIG. 2. Data display characterizing a decision rule with pure cutoffs on attributes F and G.

		$\frac{\Box \Box}{\Box} \boxed{\Box}_{c_1 p_1}$	$\begin{array}{c c} & & & \\ \hline & & & \\ & &$	$\stackrel{F_2}{\underset{C_1^{D_1}}{\boxtimes}}$	$\frac{\boxed{\square}}{\boxed{\square}} \frac{\boxed{\square}}{\boxed{\square}}_{c_2^{\mathfrak{p}_1}}$	$\frac{\boxed{\square}}{\underset{c_1 p_1}{\square}}$	$\frac{\boxed{\square}}{\underset{c_2 p_1}{\boxdot}}$
		$\frac{\boxed{2}}{\underset{c_{1}b_{2}}{\boxed{2}}}$	$\begin{array}{ c c c } \hline & \hline $		$\left \begin{array}{c c} \boxed{\square} \\ \hline{\square} \\ \hline{\square} \\ \hline{\square} \\ c_2 \\ p_2 \\ 1 \end{array}\right _1$		
$c_2$ $c_1 D$ $c_1 D$	$\begin{array}{c c} \hline & \hline & \hline \\ \hline & \hline \\ \hline & \hline \\ \hline \\ \\ \\ \\ \\$		$\begin{array}{c c} & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ &$	$\begin{array}{c c} \hline & \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\frac{\boxed{\square}}{\boxed{\square}} \frac{\boxed{\square}}{\boxed{\square}}$	$\begin{array}{c c} \hline \square & \square \\ \hline \square \\ c_1 p_1 \end{array}$	$\begin{array}{c c} \hline \square & \square \\ \hline \square & \square \\ \hline \square & \square \\ \hline \square \\ c_2^{\mathbf{p}_1} \end{array}$
	$\frac{\boxed{\square}}{\underset{\substack{c_2 p_2}}{\square}}$						

FIG. 3. Data display characterizing a mixed decision rule.

ones. Of 12 persons who first judged all 128 stimuli directly on sheets of paper, simple, complex and mixed rules were used by 5, 5, and 2 persons respectively. Looking at the second task for each subject, a slightly greater trend to simpler decision rules may be seen; 6 of 12 subjects in each condition preferred simple rules on the second task.

Despite some "simplification trend" in the second task, more than one-half of the subjects evaluated stimuli on the basis of four to five attributes, and in each group there were people who considered all seven attributes.

The data presented above lead us to a conclusion that, although the proposed task is not very easy for experts, those who were eager to analyze alternatives thoroughly managed to do this. This is borne out by the fact that the number of contradictions in answers of subjects using complex decision rules does not differ essentially from that of subjects using simpler rules. This substantiates the value of using this method for obtaining information from decision makers.

### Response Mode and Order Effects

In the course of the data analysis, the influence on decision strategies of the two different response modes and orders ('cards-sheets'' and ''sheets-cards'') was studied. We found that for the ''cards-sheets'' version, 7 of 12 subjects went from simpler ''card'' strategies to more involved ''sheet'' strategies. In the case of ''sheets-cards,'' 10 of 12 subjects went from more complex ''sheet'' strategies to simpler ''card'' strategies. In summary, it appears that cards evoke expression of simpler cutoff strategies.

### Self-Insight

We next examined how subjects' subjective estimates of the attributes' importance corresponded to their importance as inferred from actual decisions. The actual importance in the rating-sheet inquiry was established as follows: For each criterion, we calculated Z (the number of occurrences of second-level values with respect to the given attribute in alternatives classified as good ["X"]). The greater the value of Z, the less importance of this attribute in the decision rule. Calculation was followed by ordering attributes in the order of importance (low Z to high Z). Subjective importance was determined through self-report—after the inquiry and subjects gave their ordering of attributes with respect to importance for the task they had just completed.

The results are presented in Table 4 for 15 of the 24 subjects in Experiment II (self-report was not given by some subjects). These results indicate that most subjects recognize the relative importance of the various attributes in their decision strategies. The most important attributes are

Cards	Cards first; rating sheets second			Rating sheets first; cards second				
Subject number	Calculated ordering	Self-report	Subject number	Calculated ordering	Self-report			
1	BDGFECA	DBCEGFA	1	GFEADBC	GFEADBC			
3	GFDEBAC	GFEDBAC	3	FADCBGE	FEDABCG			
4	DEBCFGA	EFCAGDB	4	FGDACBE	FGDACBE			
5	DBAFEGC	DBEFGCA	5	FGCBEDA	FGCBEDA			
6	FGDABEC	FCGBEDA	6	BEFCAGD	BEFCAGD			
7	BFGCDEA	FGBECDA	10	FGCBEDA	FBEDCAG			
9	CADBEFG	CABGDFE	11	GFBCAED	GFBACDE			
11	FGBDCAE	FGDCAEB						

 TABLE 4

 Comparison between Subjective and Inferred Attribute Orderings

particularly well recognized. The main inversions in the order of attributes involve the less important attributes.

### Optimal Strategy for Testing the Decision Maker

The modeling experiments have shown that, even under random presentation of alternatives, the subjects gave only few contradictory answers and were able to use complex strategies. Hence, there is a possibility of constraining the order of stimulus presentation to reduce the time and effort involved. Larichev et al. (1978) have developed a system which enables a "border line" to be drawn between Classes X and Y on the basis of a much reduced number of judgments. The method minimizes the number of stimulus combinations presented to the decision maker in the following manner: When a combination of stimuli  $s_i \in S(i = 1, 2, ..., 2^N)$ , where S is the totality of stimulus combinations) is presented to the decision maker, classification of a number of stimuli becomes evident. Thus, if the decision maker classified stimulus  $s_i$  as belonging to Class X, combinations dominating  $s_i$  may also be regarded as belonging to this class. Similarly, if  $s_i$  is classified as belonging to Class Y, combinations which it dominates also will belong to this class. Presentation of a single stimulus from S to the decision maker will be referred to as an "experiment."

It may be naturally assumed that the number of stimuli whose classification becomes evident from the dominance condition defines the amount of information extracted from an experiment in the given point (or "informativeness" of this point). An optimal algorithm for solution of the above problem lies in carrying out a series of experiments involving the most informative points (i.e., in successively presenting to the decision maker the most "informative" estimate combinations (Yaglom & Yaglom, 1973).

The amount of information obtained depends on the outcome of the

experiment, i.e., on the classification given by the decision maker. Therefore, the algorithm minimizing the number of experiments should successively carry out experiments in those points where the amount of information will be maximal under any outcome. Denote by  $Y_i$  amount of information obtained through experiment in point  $s_i$ . Then the criterion for selection of the succession of experiments may be presented in the form of max  $Y_i$ .

It is proposed to determine  $Y_i$  through the expression

$$\Upsilon_i = \nu_x^{\ i} \kappa_x^{\ i} + \nu_y^{\ i} \kappa_y^{\ i}, \qquad (1)$$

where  $\kappa_x^i$ ,  $\kappa_y^i$  is the number of combinations whose classification (X or Y) becomes evident depending on  $s_i \epsilon X$  or  $s_i \epsilon Y$ ;  $\nu_x^i$ ,  $\nu_y^i$  are coefficients characterizing the probability that combination  $s_i$  belongs to Classes X or Y, respectively).<sup>1</sup>

Thus, each step of the proposed algorithm consists of determining the "informativeness" of all combinations whose classification was not established at the previous step and in presenting to the decision maker the maximally informative one.

We have analyzed the effectiveness of the proposed algorithm and derived upper and lower boundaries for the number of steps ( $Q_{\text{max}}$  and  $Q_{\text{min}}$ ).

$$Q_{\max} = \sum_{i=1}^{R(N-1)/2} (C_N^i - C_N^{i-1}) R \left\{ \log_2 [N - 2(i-1)] \right\},$$
(2)

$$Q_{\min} = N, \tag{3}$$

where N is the number of attributes.<sup>2</sup>

Table 5 contains values of  $Q_{\text{max}}$  and  $Q_{\text{min}}$  computed through (2) and (3). One can see that the above inquiry strategy results in significant reduction of the number of experiments needed to be faced by the decision maker.

Values of  $Q_{\text{max}}$  and  $Q_{\text{min}}$  characterize the basic limits of the algorithm steps. The structure of the algorithm is such that the number of sets depends greatly on the decision maker's preferences over the set of stimuli, i.e., on the "configuration" of the border line between the Classes X and Y. Therefore, it seems appropriate to discuss estimates of steps for particular cases.

<sup>1</sup> For more detailed description of the algorithm and, in particular, of the analytical relations for determination of the values involved in Eq. (1), see Larichev *et al.* (1978).

<sup>2</sup> To determine  $Q_{\max}$ , a theorem of chain covering of an *n*-dimension unit cube was used (see Hansel, 1966). The value of  $Q_{\min}$  was defined out of evident logical considerations (see Larichev *et al.*, 1978). Values of  $Q_{\max}$  and  $Q_{\min}$  in Table 5 were calculated supposing that decision makers' answers while determining the boundary between classes were noncontradictory.

	Number of attributes							
	3	4	5	6	7			
$Q_{\min}$	3	4	5	6	7			
$Q_{\max}$	6	10	25	41	91			
Complete selection	8	16	32	64	128			

TABLE 5VALUES OF  $Q_{max}$  and  $Q_{min}$  for Various Numbers of Binary Attributes

To analyze the proposed algorithm in actual situations, we have carried out a computer simulation of the separation into two classes of combinations of set S for N = 7. On the basis of the results of the first two experiments described above, the border line between Classes X and Y was determined for seven subjects. Next, search of these border lines by means of the algorithm was simulated. Table 6 shows the number of algorithm steps needed to determine the border line between Classes X and Y. Results obtained demonstrate that in practical problems the algorithm enables an average eightfold reduction of the number of stimuli presented to the decision maker. Of course, it is important to note that the decision maker may produce some contradictory or erroneous answers. These situations can be easily found in the process of inquiring, for we receive excess information which can be used to check the decision maker's answers. Ouestions are repeated to find the correct answer. These additional questions certainly enlarge the total number of stimuli presented to a decision maker.

### DISCUSSION

The results from these experiments confirm the proposed hypothesis, and this technique of detecting decision strategies may be used in practical decision making problems. To our mind, the decision maker's information-processing capabilities depend essentially on the task being dealt with. For example, when the attributes have just two levels, the

	Subject number							
	1	2	3	4	5	6	7	
Number of experiments required to construct the border line	16	18	24	14	11	16	12	

 TABLE 6

 Number of Steps Needed to Identify the Borderline for Seven Subjects

expert can readily evaluate the stimuli. When the attributes are scaled on more than two levels, the decision task may be much more difficult. This is also the case when the number of decision categories is increased. Being aware of this fact, one can transform in advance the decision making situation (by merging categories on the response scale, for instance) in order to enable decision makers to express their policies reliably and unambiguously. This permits us to attain one of the objectives of decision making methods, which is to assist managers in the formulation and consistent expression of their policies.

### REFERENCES

- Davis, J. M. The transitivity of preferences. Behavioral Sciences, 1958, 3, 26-33.
- Dawes, R. M., & Corrigan, B. Linear models in decision making. Psychological Bulletin, 1974, 81, 95-106.
- Hansel, G. Sur le nombre des fonctions booleennes monotones de n variables. Comptes Rendus Hebdomadaires des Seances de l'Academie des Sciences, 1966, 262, 1088-1090.
- Larichev, O. I. A practical methodology of solving multi-criterion problems with subjective criteria. In D. E. Bell, R. L. Keeney, & H. Raiffa (Eds.), Conflicting objectives in decision. New York: Wiley, 1977. Pp. 197-207.
- Larichev, O. I., et al. A method for forming scientific and technological policy in planning fundamental research. In Upravlenie Nauchnymi issledovaniyami, razrabotkami i vnedreniem novoi tekhniki. Moscow: Ekonomika Publ., 1977. [in Russian]
- Larichev, O. I., et al. Hierarchical-system method in program-oriented planning of research. Preprint of VNIISI, Moscow, 1978. [in Russian]
- Marschak, J. Decision making: Economic aspects. In International encyclopedia of the social sciences. Crowell: Collier & MacMillan, 1978. Vol. 4.
- Miller, G. A., Galanter, E., & Pribram, K. H. Plans and the structure of behavior. New York: Holt, 1960.
- Mirkin, B. G. Group choice problem. Moscow: Nauka Publ., 1974. [in Russian]
- Payne, J. W. Task complexity and contingent processing in decision making: An information research and protocol analysis. Organizational Behavior and Human Performance, 1976, 16, 366-387.
- Simon, H. A. Administrative behavior. New York: Wiley, 1960.
- Simon, H. A. The sciences of the artificial. Cambridge, Mass.: MIT Press, 1969.
- Slovic, P., & Tversky, A. Who accepts Savage's axiom? Behavioral Science, 1974, 19, 368-373.
- Tversky, A. Intransitivity of preferences. Psychological Review, 1969, 76, 31-48.

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