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GROUP DECISION IN SMALL GROUPS

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Decision support methods for group decision making in small groups are considered.

1. Introduction

Various decision support methods are available for individual decisions [1, 2]. Less developed are decision support methods for group decisions. Group decision is not restricted to voting iin elections. Common cases include decision making by committee, by jury, or by a collegiate board, all of which represent small groups of individuals. The decision maker in these cases is the entire group; we call it a decision group.

How to organize the work of a decision group? How can we guarantee that people with different preferences eventually reach an agreement? A traditional way to resolve the difficulty is by organizing meetings in which the group members act as experts evaluating different options and try to convince other group members to accept their views. These discussions often lead to a consensus opinion, which reflects a compromise among the members of the decision group. The obvious advantages of this group decision process are the following:

(i) each group member has an opportunity to express and justify his or her view;

(ii) each group member has an opportunity to listen to the opinions of all other discussion participants.

Alongside these advantages, the traditional process of group decision making sometimes suffers from the following negative phenomena:

(i) the arguments of one forceful member or several such members (a coalition) who strongly advocate the positive features of their preferred options may have an exceptional impact on the rest of the group;

(ii) the group members often waste much time in nonproductive discussions if the views of some group members are strongly divergent;

(iii) hasty application of the majority rule excludes the opinions of some group members from general consideration.

Various approaches have been developed in an attempt to deal with the negative features of traditional group decision making. In what follows, we consider the choice problems that have to be solved by a decision group: certain options are presented to the group (investment options, shares of stock, etc.); it is required to identify the best options, to divide the options into groups, etc., i.e., to solve some typical individual-decision problems.

Group decision making is radically different from individual decision making. Each group member usually has his or her own view of the situation. If these views are identical, or if the group includes a dictator who imposes his preferences on the rest, then the group decision problem does not arise. In general, however, the main task of a decision group is to find a compromise acceptable to all group members.

There are three directions of research in small group decision making:

1. Non-zero-sum games. This is one of the lines of research in game theory [3] designed to develop mathematical models that describe the search for a compromise, the determination of equilibrium points. Research in this direction is purely theoretical in most cases.

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2. *Group decision support systems.* Local networks are developed for group members, as well as formal algorithms for comparison of preferences on a given set of options. Decision support systems are generally intended for familiarizing each group member with the views and opinions of the rest of the group. The problem of reaching a consensus is either not considered or is reduced to simple averaging of preferences. In practice, this approach does not lead to responsible decision making.

3. *Organizing group discussions with the help of a facilitator (expert or consultant).* This approach appears to be the most promising in practice. Decision conferences are the best known example in this direction. The decision conference approach is connected with the names of C. Camerer (USA) and L. Phillips (England). They were the first to develop the methodological foundations of decision making in this framework and have been successful both practically and commercially.

2. Organization and Implementation of Decision Conferences

Assume that the decision group is the board of directors or a large corporation. The corporation is faced with difficulties: the market share is dropping, the profits are declining. Something has to be done. Some of the directors propose to launch a new product, other directors propose to increase the advertising budget, yet others propose to upgrade the existing products.

To develop an agreed strategy, the president decides to invite a consultant, a so-called facilitator who specializes in decision conferences. The facilitator visits the corporation, talks to some of the directors, and familiarizes himself with the situation. Of course, the facilitator is unable to acquire the same depth of knowledge of the manufacturing and financial problems as the directors. But he attempts to master the business language used by the directors in their discussions, to understand the power and influence structure among the directors.

After a couple of weeks a decision conference is scheduled. It is conducted in a special room equipped with appropriate technical facilities: a computer, projection screens to display the computer output, a projector, whiteboards, marker pens, etc. The conference is organized for 1–2 days (usually over the weekend), and during this time the directors focus only on strategy, abandoning all routine concerns. The conference is run by the facilitator. He recognizes speakers representing different points of view. He controls the discussion process, guiding it along a constructive path; he himself poses questions, attempting to elucidate the strong and weak aspects of the proposed options. The facilitator attempts to highlight the commonalities among the various options, stressing the elements of agreement. He attempts to help the conference participants to find mixed strategies (if this is possible) without losing face, to drop options whose shortcomings have become glaringly obvious. The facilitator suggests to evaluate the options by various criteria simultaneously. If there are differences in evaluations, the facilitator tests the sensitivity of choice to these differences, and so on.

The success of a decision conference largely depends on the facilitator's expertise. The facilitator should have certain personal qualifications: ability to quickly grasp the essence of the problem and the existing disagreements, ability to quickly be accepted as "one of the team" in the course of discussion, etc. The facilitator should have extensive practical experience and should be able to channel the discussion correctly. Like an experienced orchestra conductor, he should "wave the baton" at the right time, giving the floor to that participant who at the particular juncture can have the most productive impact on the discussion.

These skills cannot be learned from a textbook, and indeed no such textbooks exist. There are scattered publications [4] which use imaginary cases to demonstrate the main ideas of decision conferencing. The success of decision conferences has impressive results on corporate activity. Even relative success can influence strategic choices. On the other hand, failure of a decision conference also has a strong, but negative, effect, strengthening the disagreements and divergent opinions.

3. Method to Organize the Work of a Decision Group

A method to organize the work of a decision group in the process of a decision conference was developed and applied in 1975 at the initiative of S. V. Emel'yanov, then Deputy Director of the Institute of Control Problems and Corresponding Member of the USSR Academy of Sciences [5]. The method relies on the following considerations.

1. Effective discussion of a problem by a decision group requires preliminary analysis to elucidate agreements or disagreements of group members concerning the advantages and shortcomings of the relevant options.

2. Preliminary analysis should identify the particular questions on which the group members agree or disagree, elucidate the emergent consensus, highlight the emerging coalitions.

Experience with organization and implementation of surveys suggested the exceptional importance of dividing the question under discussion into separate component parts. When options are compared and their quality is evaluated, it is highly important to separate the components of quality — the various relevant criteria — and to evaluate the options by each criterion separately. This approach achieves the following goals:

I. Higher objectivity of extracted information. When options are evaluated as one whole, there is a much greater scope for tendentious, subjective evaluation. Focusing on the positive qualities of the option, the group members often unintentionally (or perhaps intentionally) ignore the negative qualities. When options are evaluated by multiple criteria, each group member largely acts as a qualified expert, an "observer" measuring the qualities of the objects on common scales used by all group members. Conscious upward or downward biasing of these partial evaluations may damage the reputation of a qualified, knowledgeable expert (when these evaluations are checked and discussed by other experts). On the other hand, a global evaluation of the whole option always can be explained by the expert as the outcome of a certain weighting of component qualities.

II. More concrete and transparent discussion of the extracted information. When group members discuss the evaluation obtained by a single criterion, the disagreements are typically smaller than when discussing a global evaluation of the entire option. It is easier to obtain additional information on a narrowly formulated question, easier to collect undisputed facts.

3.1. Preliminary Stages. The following sequence of actions can be proposed.

I. Establish a list of criteria. Identify the group of relevant criteria that should be used for evaluating the given set of options. The list of these criteria is generated by questioning each group member. The list of criteria collected from all group members is then cleared with each group member individually. To help the group members, the facilitator may propose a tentative list of criteria which are useful for similar questions.

II. Develop evaluation scales for each criterion. Based on the accepted terminology of the particular decision group, the facilitator develops for each criterion a scale of several verbal qualitative ranks arranged from best to worst. These scales are then agreed upon with each group member. The scale ranks should be clearly understandable and free from ambiguity.

III. Collect information. The group members receive blank forms listing all the criteria with the corresponding scales. The number of blanks is equal to the number of options. Each group member evaluates the options by each of the criteria, assigning one of the ranks on the relevant scale to the given option. If necessary, each group member may request (through the facilitator or on his own) additional information that is needed for evaluating the option.

The data collected through this individual survey of group members carry information about the degree of agreement or disagreement of their views. This information is extracted by special analysis.

3.2. Analysis of Information. It is desirable to have an analytical technique that estimates the degree of agreement among group members. This technique should identify the options for which the evaluations show the greatest disagreement and the criteria that produce the greatest divergence in opinions among group members.

The "projection" method reducing the dimension of the data space offers an analytical technique that, in our view, satisfies these requirements [6, 7]. Dimension reducing methods are part of factor analysis [8]. The main idea of "projection" methods can be described as follows. Consider an r-dimensional space with an n-point configuration. It is required to "project" these points into a subspace of lower dimension (usually two- or three-dimensional space) with minimum (in the sense of some criterion) distortion of the distances between the options as measured in the original space.

"Projection" methods, like other methods of factor analysis, assume that there exists a small number of generalized factors characterizing the main features of the objects. This assumption is verified by applying the method and examining its outcomes.

In our setting, each option is a point in the criterion space. Introducing a certain metric, we can place in the same space the points the characterize the attitude of the group members to the given options. Then "projection" methods can be applied to represent the options and the views of the group members in a plane so that

(i) the distance between group members characterizes the degree of agreement among them: small distances correspond to agreement of views;

(ii) the distance between a group member and an option characterizes the evaluation of the option by the group member: small distances correspond to options that receive high evaluations;

(iii) the distance between options corresponds to the degree of their "similarity" based on evaluations by group members.

The resulting two-dimensional distribution of options and experts in the plane provides a general picture of the degree of agreement among the group members as a whole and for each option separately. The opinions of the group members naturally may show considerable divergence in many cases.

The proposed approach successively applies the "projection" method to "robustify" the initial information by successively eliminating some criteria, options, and evaluations and observing the degree of agreement among group members given the remaining information. In this way, we can identify the disputed questions (criteria, estimates, options) and focus the attention of group members on these questions for further discussion.

Although simultaneous "projection" of options and experts has been used previously (in individual scaling problems, see, e.g., [9]), our problem has some distinctive features. Evaluation of options by multiple criteria rapidly increases the number of constraints imposed on options and experts during "projection." We are thus faced with the real danger of introducing large distortions in the initial distances between options in the many-dimensional space.

The following approach may prove helpful in this respect.

I. First consider only the distances between the group members. They characterize the degree of agreement among the group members evaluating the options by multiple criteria.

II. Solve the "projection" problem only for points representing the group members in accordance with the specified distances. Efficient mapping is achieved using the criterion

$$S_1 = \frac{1}{\sum\limits_{\substack{m,e \\ m<e}}^{N_e} D_{me}} \sum_{\substack{m,e \\ m<e}}^{N_e} \frac{(D_{me} - d_{me})^2}{D_{me}}, \tag{1}$$

where D_{me} are the initial distances between points m and e, d_{me} is the distance obtained in the current step of solving the problem, N_e is the number of group members (experts). Minimization is performed by the conjugate gradient method [10].

III. Successively "robustify" the initial information step by step (see below) until a satisfactory agreement among group members is achieved or until one or several coalitions emerge. The degree of agreement is established by analyzing the "projections" on appropriate diagrams.

IV. When agreement has been achieved between group members or coalition members (at the cost of removing some of the information), distances are introduced between experts and options and the "projection" problem is solved again simultaneously for all options and all group members (or coalition members). It is assumed that the previous stages (partial removal of information) have ensured sufficient proximity of the points representing the group members, and therefore "projection" does not involve excessive distortions.

Simultaneous projection uses the criterion

$$S_2 = \frac{1}{\sum\limits_{e,j} D_{ej}} \sum_{e,j} \frac{(D_{ej} - d_{ej})^2}{D_{ej}}, \tag{2}$$

where D_{ej} is the initial distance between points e and j, d_{ej} are the Euclidean distances between pairs of points corresponding to projections of options and experts. Minimization is again performed by the conjugate gradient method [10]. The average distance of an option from a cluster of group members with identical opinions determines its location in the final ordering of options by the opinion of this cluster. Let us enumerate the stages used to perform the partial removal ("robustification") of information.

1. First define a simple metric distance between experts:

$$D_{e_1 e_2} = \left[\frac{1}{N_0} \sum_{j=1}^{N_0} \frac{1}{N_k} \sum_{k=1}^{N_k} \frac{(z_{jk}^{e_1} - z_{jk}^{e_2})^2}{N(k)} \right]^{1/2}, \qquad e_1 e_2 = 1, 2, \ldots, N_e, \tag{3}$$

   where z_{jk}^{e} is the evaluation of object j by expert e using criterion k, N_e is the number of experts, N_0 is the number of options, N_k is the number of criteria, $N(k)$ is the number of evaluations on the scale of criterion k, e_1 and e_2 are the indices of the experts.

2. Define a distance that allows only for agreement of expert opinions concerning the preference of one option over other options by each of the criteria:

$$D_{e_1 e_2} = \frac{2}{N_0(N_0 - 1)} \sum_{\substack{i,j \\ j_1 < j_2}}^{N_0} \frac{1}{N_k} \sum_{k} \begin{cases} 0, & \text{if } (z_{ik}^{e_1} - z_{jk}^{e_1})(z_{ik}^{e_2} - z_{jk}^{e_2}) < 0, \\ 1, & \text{if } (z_{ik}^{e_1} - z_{jk}^{e_1})(z_{ik}^{e_2} - z_{jk}^{e_2}) > 0, \end{cases} \tag{4}$$

   where i, j are option indices.

3. If several evaluations are given on some criterion scales, then these evaluations are "pooled," i.e., we attempt to combine the different evaluations into two extreme evaluations characterizing the presence and absence of a quality by the given criterion.

4. To elucidate the criteria that produce the greatest disagreement among experts, construct for each pair of experts the matrix

$$\delta_{ij} = |\delta_{ij}^{N_1}, \delta_{ij}^{N_2}, \ldots, \delta_{ij}^{N_k}|, \tag{5}$$

where

$$\delta_{ij}^{N_k} = \begin{cases} 0, & \text{if } (z_{ik}^{e_1} - z_{jk}^{e_1})(z_{ik}^{e_2} - z_{jk}^{e_2}) < 0, \\ 1, & \text{if } (z_{ik}^{e_1} - z_{jk}^{e_1})(z_{jk}^{e_2} - z_{jk}^{e_2}) > 0. \end{cases} \tag{6}$$

We define the coefficient T_k characterizing the degree of disagreement of the experts by criterion k:

$$T_k = \frac{2}{N_0(N_0 - 1)} \sum_{\substack{i,j \\ i<j}}^{N_0} \delta_{ij}^k \,. \tag{7}$$

The criteria are ranked by the coefficient T_k and are successively removed from the queue of those waiting to be examined.

In each of the stages listed above we analyze a sequence of diagrams representing the "projection" results. When a cluster of points representing the group members is observed to be sufficiently tight, the process of information removal stops. If no tight clustering is achieved after removing half the criteria in stage 4, the outcome of the analysis establishes the existence of deep irreconcilable disagreements among the group members. Such a case is fairly rare in practice. If the analysis fails to detect overall agreement among all group members, it identifies the existence of coalitions.

Having identified a tight cluster of group members, we define for this cluster a metric distance between an option and a group member:

$$D_{ej} = \left[\frac{1}{N_k} \sum_{k=1}^{N_k} (z_{jk}^e)^2 \right]^{1/2} . \tag{8}$$

The ranking of options produced by projection characterizes the average opinion among the members of the particular cluster. The degree of agreement within the cluster may be estimated by the Kendall coefficient of rank correlation [11], comparing the ranking of the cluster members with the overall ranking.

3.3. Decision Conference. The results of the analysis are communicated to all group members during the first discussion of the question. These results are conducive to a more efficient organization of the work of the decision group: the group first discusses the disagreements by separate criteria, requesting additional information if needed, etc. The results of analysis presented in a convenient visual format help to guide the group discussion.

3.4. Practical Example. The proposed method was applied to organize the work of a committee. The task of the committee included classifying the research projects submitted for a competition into two groups: good (first prize) and bad (second prize). The evaluation questionnaire is reproduced below. Figure 1 shows the initial projection using the distance (3). Figure 1 clearly shows an emergent agreement among 7 out of the 8 group members, and three of them (3, 6, 7) actually form a tight cluster. Then the evaluations were "pooled," and the result is shown in Fig. 2.

The procedure produced virtually complete agreement among most committee members. This result was reported by the facilitator before starting the discussion of the specific projects. The facilitator also announced that only one group member (5) diverged from the rest of the group in his evaluation of four projects by three criteria. During the discussion this member was convinced to revise his original evaluations. The discussion took much less time than similar committee meetings, and the emergent majority opinion was adopted by consensus.

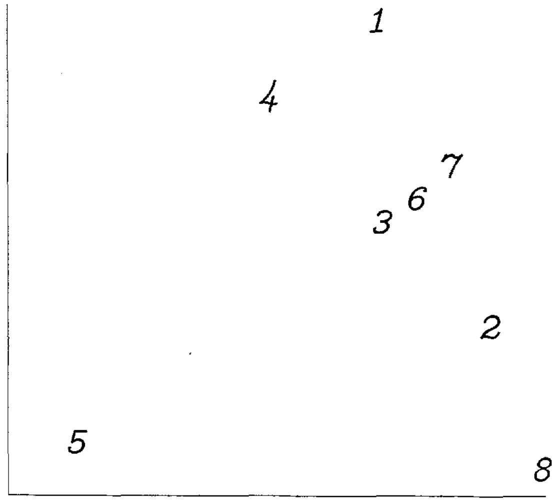

Fig. 1

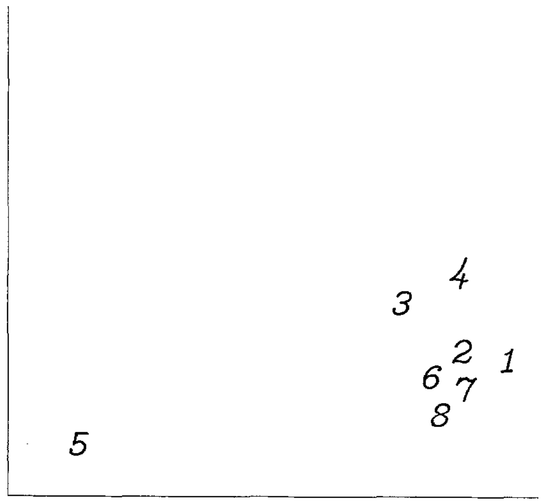

Fig. 2

QUESTIONNAIRE

1. Novelty. The results (or the main ideas) of the project were first published (or reported)
   - a) not earlier than 1971;
   - b) in 1968–1971;
   - c) earlier than 1968.
2. Value of the new regularities discovered by the author:
   - a) the results of the study open possibilities for solving a wide range of important scientific problems;
   - b) the results of the study make it possible to solve a number of new scientific problems;
   - c) the study provides a new explanation of known scientific facts;
   - d) the scientific value of the new regularities is unclear.

3. Originality of the method (procedure, implementation). The method (procedure, implementation) proposed by the author is
   a) highly original;
   b) original;
   c) not very original;
   d) the originality of the method (procedure, implementation) is not clear.
4. Comparison with best available method (procedures, implementation) by main parameters. The proposed method (procedure, implementation) has
   a) significant advantage over existing methods;
   b) modest advantage;
   c) no significant advantage;
   d) is inferior to existing methods.
5. Breadth of application of the results. The results may be used by
   a) a wide range of experts, both practitioners and theoreticians;
   b) researchers specializing in a particular area of control theory or practice;
   c) a narrow range of experts.
6. Value for national economy:
   a) the results of the study have led to solution of some problems on the level of national economy, as confirmed by official documents;
   b) there are plans to operationalize the results of the study;
   c) scope for operationalization of the results is not clear.

4. Conclusion

Group decision making in small groups, such as committees, jurors, etc., is a common phenomenon in practice. Effectiveness of such groups can be substantially improved by decision support methods, analysis of the opinions of group members, and efficient organization of how the group works.

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