QUALITATIVE COMPARISON OF MULTICRITERIA ALTERNATIVES

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A new method of multicriteria alternatives ranking adapted to the possibilities and limitations of human information processing system is presented in the paper. On the basis of decision maker preferences, the estimations on discrete, ordinal scales of criteria are ordered on a joint scale. The pair-wise comparison of multicriteria alternatives is constructed with the help of this joint ordinal scale. There are three possible outputs of alternative comparison: preference, indifference and incomparability. A new approach of estimation of a decisive power for such decision rule is given. The comparison of quantitative and qualitative methods of measurements in decision analysis is done.

1 Introduction

The great value of Habitual Domain approach consists in giving human dimension to decision problems [1]: how to help people find new, promising perception of a decision problem.

A different way to introduce the human aspects into the world of decision making is to take into account the possibilities and limitations of the human information processing system. The approach of Verbal Decision Analysis is oriented to this goal [2]. It is developed for the solution of the so-called unstructured problems [3], for which the qualitative factors play the major role.

The method ZAPROS [4] from the family of Verbal Decision Analysis methods is developed for the task of multicriteria alternative ranking.

The positive features of this method consist in the following:

- the utilization of verbal descriptions of qualitative factors from the very beginning to the end of problem analysis without unnecessary transformation of such factors into quantities;
- the utilization of a psychologically valid approach for preferences elicitation from DM;

- the constant check of DM information for contradictions and, as a consequence, low sensitivity to human errors;
- the possibility for gradual development of a decision rule for DM.

The output of ZAPROS is the partial order for that each two alternatives could be in the one from three relations: preference, indifference and incomparability. The relation of incomparability arises due to a limited scope of information required from the decision maker in framework of ZAPROS. The procedure of information elicitation corresponds to restricted human ability to process multidimensional information. But the limited (though psychologically valid) information elicited from the decision maker does not allow to construct a complete order of multicriteria alternatives.

The important problems remain: how "powerful" is the partial order constructed by ZAPROS? How many alternatives could be in the relation of incomparability?

This paper is devoted to the analysis of this problem. Below we give a practical problem of ZAPROS application, the main ideas of the method. We present Joint Ordinal Scale as a qualitative technique for alternatives pair-wise comparison. We present a new approach to estimating the maximum possible number of incomparable alternatives. The results give the direction to development of psychologically valid and decisive analytical methods.

2 Practical Task

The typical practical task for ZAPROS application is the choice of projects that could be submitted to a Fund created for the support of best projects. There are many funds of such kind in different countries. Usually, after an announcement of project competition, organizers are to develop a rule for the evaluation of submitted projects. The criteria used for the evaluation reflect a policy accepted by the organizers. Experts are usually invited to nominate for each project evaluations by the criteria.

But the set of criteria is not sufficient for expression of the Fund policy. A decision rule based on such criteria is needed.

It is not known in advance, which projects having the estimate combinations given by experts would be submitted to the Fund. But in any case it is necessary to rank-order the submitted projects according to their overall quality. Each project requires some resources. Given ranking of projects, it is easy to select a group of best projects within the limit of available resources.

Our experience demonstrates that ZAPROS is a reliable method for the solution of practical tasks of such kind [2].

3 The Main Ideas of ZAPROS Method

The problem may be formulated as follows:
Given:
1. $K = 1,2,...,N$ is a set of criteria;

2. n_q is the number of possible values on the scale of the q-th criterion $(q \in K)$;

3. $X_q = \{x_{iq}\}$ is a set of values for the q-th criterion (the scale of the q-th criterion); $|X_q| = n_q$ $(q \in K)$; the values on a scale are ordered from the best (first) to the worst (last); the order of the values on one scale does not depend on values on the others. Each criterion has an ordinal scale of estimations with verbal expressions of quality degrees. Such expressions are pieces of the natural language used by decision maker (DM) and experts in everyday life.

4. $Y = X_1 * X_2 * \ldots X_N$ is a set of vectors $y_i \in Y$ of the following type $y_i = (y_{i1}, y_{i2}, \ldots, y_{iN})$ where $y_{iq} \in X_q$ and $P = |Y| = \prod_{i=1}^{i=N} n_i$;

5. $A = \{a_i\} \in Y ; i = 1,2 \ldots t$ -the set of t vectors describing real alternatives.

Required:
To rank multicriteria alternatives on the basis of a decision-maker's preferences.

The method of DM's preference elicitation consists in pair-wise comparison of two estimations taken from two criteria scales for each pair of criteria, by supposition that there are the best estimations on the others. It was shown in the experiments that people could give reliable answers to such questions.

"Please, compare two estimations from two criteria scales

x_{if} and x_{jk}

(x_i, x_j -are the estimations on the scales of criteria f and k)

and select one from the following answers:
1. The first estimate is better than the second;
2. The estimates are equal in quality;
3. The second estimate is better than the first,

supposing that there are best estimations for other criteria."

The answers of DM allow to rank all estimations from the scales of two criteria. This ranking could be called Joint Ordinal Scale for two criteria.

Let us suppose that we have two criteria with ordinal scales having three estimations arranged from the best(first) to the worst(third).

x_{11}, x_{12}, x_{13} - estimations on the scale of the first criterion.

x_{21}, x_{22}, x_{23} – estimations on the scale of the second criterion.

The possible Joint Ordinal Scale for two criteria is given below.

$$x_{11}, x_{21} \Rightarrow x_{12} \Rightarrow x_{22} \Rightarrow x_{13} \Rightarrow x_{23}$$

Evidently, the best estimations on the criteria scales x_{11}, x_{21} are equally good for the decision maker. The pair-wise comparisons of other estimations give the Joint Ordinal Scale for two criteria.

Next, different pairs of criteria are taken for estimations comparison by supposition that the estimations for others are best.

There are 0.5N(N-1) possible pairs of criteria. The preferences of DM are elicited for each pair. So, 0.5N(N-1) rankings of estimations for all pairs of criteria could be constructed.

Having Joint Ordinal Scales(JOS) for all pairs of criteria, it is possible to construct JOS for the estimations of all criteria[2,4]. The necessary condition is one of preference independence for all pairs of criteria. The check for preference independence consists in pair-wise comparison of the same estimations from scales of two criteria by supposition that there are worst estimations on the other criteria. In the case of criteria dependence, the verbal description of a problem is to be changed to achieve criteria independence (see details in [2]).

Let us suppose that the criteria are independent. For a ginen case, the possibility to use JOS for pair-wise alternatives comparison was proved.

According to the statement of the problem, a set of alternatives having evaluations on multiple criteria is given.

Let us note as the function of alternative quality-$V(y)$ and make the following supposition about the properties of this function:

-There are maximum and minimum values of *V(y);*

-For independent criteria, the value of *V(y)* is increasing when the evaluations on each criterion are improving.

It is possible to find a corresponding place on JOS for each evaluation of an alternative (a component of a vector $y_i = (y_{i1}, y_{i2}, \dots, y_{iN})$) and find a rank of each evaluation according to JOS.

For each alternative it is possible to define the corresponding vector of components ranks on JOS:

$$V(y_i) \Leftrightarrow V(r_k, r_t, \dots, r_g)$$

$V(y_j) \Leftrightarrow V(q_s, q_d, ..., q_m)$

where:

$r_k, r_l, ...r_g$ - ranks of components for the vector $y_i = (y_{i1}, y_{i2}, \ldots, y_{iN})$.

$q_s, q_d, ..., q_m$ - ranks of components for the vector $y_j = (y_{j1}, y_{j2}, ..., y_{jN})$.

The following statement(Statement 1) is proved:

If the condition of independence is true for all pairs of criteria and the ranks of the components for y_i are no worse than the ranks of the components for y_j and at least for one component of y_i the rank is better, then the alternative y_i is more preferable for DM in comparison with y_j, and $V(y_i) \succ V(y_j)$.

If the components of both vectors have the same ranks, the vectors are equal in quality. If the conditions of preference and equivalence are not true, the alternatives y_i and y_j are in the relation of incomparability.

Let us suppose that there are only two criteria with the estimates located on JOS presented above. There are alternatives with the following evaluations:

$A1(x_{11}, x_{21})$; $A7(x_{12}, x_{23})$; $A8(x_{13}, x_{22})$; $A9(x_{13}, x_{23})$.

According to JOS, the alternatives are in the following relations:

$A1PA7$; $A1PA8$; $A1PA9$; $A7NA8$; $A7PA9$; $A8PA9$,

where: P-relation of preference and N-relation of incomparability.

On the basis of a binary relation between the alternatives, it is possible to construct a partial order on the set of alternatives.

Let us single out, on the basis of binary relations all nondominated alternatives and refer to them as the first nucleus. After removing the first nucleus, let us select the second, and so on.

The rule of rank assignment to an alternative is as follows. The alternatives from the first nucleus have rank 1. An alternative is ranked «i» if it is dominated by an alternative ranked « i-1» and itself dominates an alternative ranked «i+1».

If an alternative is dominated by an alternative ranked «i» but itself dominates an alternative ranked «i+j», then its rank is fuzzy within the range from «i+1» to «i+j-1».

In our example, a1 has rank 1, a2 and a3 have rank 2, and a4 has rank 4.

The ranks nominated in the way defined above could be called relative because they are related to the given group of alternatives. But using the same algorithm it is possible to assign an absolute rank to every alternative. The ranks can be called absolute if they are related to all possible alternatives from set Y (all possible combinations of evaluations upon the criteria).

The main ideas of ZAPROS method presented above demonstrate that the pair-wise alternatives comparison is based on JOS. JOS allows one to use qualitative, logical comparison of alternatives without utilization of any numbers. But the **decisive power** of the method depends on how often the incomparability condition appears as the result of such comparisons.

4 The Decisive Power Of JOS for Binary Criteria Scales

For the particular case of the ordinal scales with two estimates, analysis was made to evaluate the possible number of incomparable alternatives as the result of comparisons based on JOS[5].

The following approach was chosen:

1. The general number Q of alternative pairs is calculated:

$Q = 0.5x2^N(2^N - 1)$.

2. The number of alternative pairs that are always in dominance relation due to the properties of ordinal criteria scales is defined by the formula:

$$D = 2^N - 1 + \sum_{j=i}^{N-1} C_N^j (2^N - 1).$$

The difference $B = Q - D$ defines the number of potentially comparable alternative pairs- PCA pairs, for whose the result of pair-wise alternatives comparison depends on the decision maker's preferences.

3. Taking JOS for second estimations from criteria scales, the formulas have been developed to evaluate the consequences of JOS relations for arbitrary pairs of alternatives. The alternatives have been allocated into Pareto layers and calculations were made separately for each layer and between the elements of the layers. Finally, the number of remaining PCA pairs - S, that could not be compared on the basis of JOS (incomparable alternatives) have been found. The result is given by the following table:

Table 1.

N	3	4	5	6	7	8	9	10
S	1	6	26	132	485	2363	7861	37505
B	9	55	285	1351	6069	26355	111645	465751
S/B	0.11	0.10	0.09	0.10	0.08	0.09	0.07	0.08

The results presented by table 1 demonstrate that for the case of binary scales, JOS is really powerful tool for alternatives comparison, because not less than 90% of alternative pairs could be compared.

This result is based on special properties of binary scales. Unfortunately, we could not use the same approach for the scales with three or four estimations. The practical tasks with criteria scales having three or four estimations are widespread in decision making. That is why, a different approach is needed for the evaluation of JOS **decisive power** in general case.

5 Special Order of Alternatives

Let us take for two alternatives the vectors of ranks corresponding to the places of components on JOS.

$$V(y_i) \Leftrightarrow V(r_k, r_t, \dots, r_g)$$

$$V(y_j) \Leftrightarrow V(q_s, q_d, \dots, q_m)$$

It is possible to order the ranks of components beginning from the best and create the vectors of ordered ranks.

$$V(y_i) \Leftrightarrow W_i(r_{i1}, r_{i2}, \dots, r_{iN})$$

$$V(y_j) \Leftrightarrow W_j(q_{j1}, q_{j2}, \dots, q_{jN})$$

In accordance with Statement 1 (see above), the following algorithm could be used for the comparison of any two alternatives.

1. The ranks r_{i1} and q_{j1} are compared. Let us suppose that one of them is bigger (therefore, worse in accordance with JOS).

$$r_{i1} > q_{j1}$$

If in pair-wise comparison of other ranks, the ranks of W_i would be bigger or equal to the ranks of W_j, then:

$$V(y_j)\, P\, V(y_i).$$

2. If at least in one comparison of the ranks,

$$r_{ik} < q_{jk}$$

alternatives are in the incomparability relation:

$$V(y_j)\, N\, V(y_i).$$

In accordance with the algorithm, it is possible to suggest the following method of investigating **decisive power** of JOS in a general case.

First, all possible alternatives y_i (all combinations of criteria estimates) are allocated in special order-SO, related to their estimates on JOS. Let us assign rank 1 to the best estimations on criteria scales (first point on JOS). The rank 2 is given to the next estimate on JOS and so on, going from the best to the worst estimations.

The first alternative in a special order would be with all components having rank equal 1, the second –all 1 except 2 in the last place, the third-all 1 except 3 in the last place and so on. Speaking differently, in the beginning of the special order there are alternatives with one evaluation different from the best and ordered according to the sequence of the estimation on JOS. The next group of alternatives has two evaluations different from the best ones. The pairs of such evaluations are created from JOS as the pairs of estimations on JOS ordered by the quality (12, 13, 14,....23, 24,....(t-1)t) where t is the number of estimation on JOS. In the same way, the groups of alternatives having 3,4,...(t-1), t evaluations are created from JOS ordered by the quality.
The following properties of SO could be proved.

Statement 1
The upper alternative in SO can be only in P and N relations with alternatives of a lower level.

Proof. By the construction, the alternatives in one group are arranged by the decreasing quality. An alternative from the lower group cannot be better in the comparison with one from the former group due to the fact that it has a larger number of the digits different from 1.
Therefore, an alternative from the lower level of SO cannot dominate an alternative from the upper level.

Statement 2
By comparison of two vectors from SO, the same numbers located in any digit of the vectors can be cancelled.

Proof. The same numbers in two vectors denote the same evaluations on JOS. In the case of criteria independence they can be eliminated during comparison.

Let us take two arbitrary alternatives from SO and make the comparison of numbers in digits from the corresponding W_i vectors beginning from the last digits.

Statement 3
If a number in a digit of upper vector is bigger than a number in the same digit of a lower level vector, then the corresponding alternatives are incomparable.

Proof. In the case of domination, a number in a digit of dominating alternative is bigger or equal to the corresponding number in the same digit of dominated alternative. Therefore, in the opposite case, there are incomparable alternatives.

Statements 1, 2 and 3are the basis of the method for the evaluation of JOS **decisive power**. SO created in the way described above is used for the successive sets of alternatives comparisons beginning from the top of SO. In such comparisons incomparable alternatives are calculated.

Let us demonstrate the proposed method on the example given above.

In the case of two criteria with three estimations on scale, the JOS and corresponding ranks of estimations are:

$$\underset{1}{x_{11}, x_{21}} \Rightarrow \underset{2}{x_{12}} \Rightarrow \underset{3}{x_{22}} \Rightarrow \underset{4}{x_{13}} \Rightarrow \underset{5}{x_{23}}$$

The Special Order is given in table 2.

Table 2

N of alternative	Alternative	Vector W_i
A1	x_{11}, x_{21}	1,1
A2	x_{21}, x_{12}	1,2
A3	x_{11}, x_{22}	1,3
A4	x_{21}, x_{13}	1,4
A5	x_{11}, x_{23}	1,5
A6	x_{12}, x_{22}	2,3
A7	x_{12}, x_{23}	2,5
A8	x_{22}, x_{13}	3,4
A9	x_{13}, x_{23}	4,5

Let us stress that vectors 2,4 and 3,5 are forbidden due to the fact that any alternative has evaluations on both criteria.

Using the order of alternatives given by Table 2, it is easy to calculate the number of incomparable alternatives. The general number of alternative pairs is 36. The pairs (4-6), (5-6), (5-8) are in N relation.

SO is also a convenient presentation to find the absolute ranks(see above) of all alternatives. Using the algorithm given above, we receive the following ranks.

Table 3

Altern	A1	A2	A3	A4	A5	A6	A7	A8	A9
Rank	1	2	3	4	5	4-5	6	5-6	7

We could compare the ranks in table 3 with relative ranks given above to three alternatives.

The existence of incomparable alternatives gives as the consequence, alternatives with fuzzy ranks and makes a ranking less definite. That is why it is important to evaluate the possible number of incomparable alternatives.

The method of evaluation JOS decisive power has been implemented in a computer program. The results for three estimations on every criterion scale (the case is widespread in the practice) are given in Table 4.

Table 4

N	2	3	4	5
Q	36	351	3240	29403
S	3	74	956	10692
S/Q	0.08	0.21	0.29	0.36

In Table 4 Q is a general number of alternative pairs, S-number of remaining PCA pairs.

We could observe that by $n_q = 3$ the number on incomparable alternatives increases rapidly with the increasing of the number of criteria. The increasing would be sharper by a bigger number of the estimations on the criteria scales. A different way of qualitative problem analysis is needed.

6 JOS for The Problems of Big Size

Evidently, the size of the problem is defined by

$$P = |Y| = \prod_{i=1}^{i=N} n_i$$

In the case of the problems with a large number of alternative pairs, a different way of analysis is needed. Really, in practical situations, DM wants first to investigate the most essential features of a problem. That is why the information from DM could be elicited in a different way.

The question posed to DM is as follows:
"Please, compare two estimations from two criteria scales

x_{if} and x_{jk}

(x_i, x_j -are the estimations on the scales of criteria f and k)

and select one from the following answers:

6 The first estimate is much better than the second.
7 There is no big difference in the alternative values.
8 The second estimate is much better than the first."

The answers of DM allow one to perform the ranking only for the cases of strong preference in the comparison, to rank the estimations from the scales of two criteria that are in evident superiority relation. This ranking could be called as Strong Joint Ordinal Scale (SJOS) for two criteria.

The general SJOS for the estimations of all criteria could be constructed exactly in the same way as general JOS.

According to our experience, in multiple criteria practical situations the big part of alternative pairs includes the alternatives which are not very different by value for DM. Very often only one or two estimations on criteria scales are of much greater (or lesser) importance for DM. That is why an analysis of SJOS **decisive power** could be made by supposition that the majority of estimations has approximately the same value for DM.

As a basis for this analysis we take SJOS for that second, third (and others) estimations on criteria scales have the same value for DM. For our example, the joint scale in such case is:

$x_{11}, x_{21} \Rightarrow x_{12}\ x_{22} \Rightarrow x_{13} x_{23}$

Giving the rank from 1 to 3 to three points on SJOS, it is possible to create SO in following form:

Using the order of alternatives given by Table 5, it is easy to calculate the number of incomparable alternatives. The general number of alternative pairs differing in value is 15. The pair (3-4) is in N relation.
For SO given in table 5 it is possible to define the absolute ranks of the alternatives. There are no fuzzy ranks in this case.

Table 5

N of alternative	Alternative	Vector W_i
B1	x_{11}, x_{21}	1,1
B2	$x_{11}, x_{22}; x_{21}, x_{12}$	1,2
B3	$x_{11}, x_{23}; x_{21}, x_{13}$	1,3
B4	x_{22}, x_{12}	2,2
B5	$x_{12}, x_{23}; x_{22}, x_{13}$	2,3
B6	x_{13}, x_{23}	3,3

Table 6

Alternat.	B1	B2	B3	B4	B5	B6
Ranks	1	2	3	3	4	5

To evaluate the **decisive power** of SJOS, the results for three estimations on every criterion scale are given in Table 7

Table 7

N	2	3	4	5
Q	15	45	105	210
S	1	5	13	35
S/Q	0.06	0.11	0.12	0.16

The number of incomparable alternatives depends on the number of points on joint ordinal scale. In the experiments, the number of points on SJOS was much smaller than the number of points on JOS. That is why table 5 gives the evaluation of SJOS decisive power close to the real one.

The comparisons of alternatives made with the help of SJOS define a more evident difference between the alternatives. If some real alternatives within the group are equivalent after such comparison, a more detail comparison based on JOS could be used only for this group. Therefore, such procedure saves efforts of DM.

7 Practical Importance of Verbal Measurement

The experimental study was made to compare verbal and numerical methods of decision making[6]. The subjects were college students of Texas A&M University nearing graduation, who were in job search process, facing opportunities similar to those given in the study.

Let us suppose that a college graduate has several offers (after interviews) and he (or she) is to make a decision. Every variant is acceptable, but of course, one variant is better in one aspect and the other – in another. So, the student has a multicriteria problem. The student was asked to solve it with the help of the appropriate multicriteria method.

Let there be Q criteria, by which N alternatives are evaluated. Each alternative y_i corresponds to the vector $y_i = (y_{i1}, y_{i2}, \ldots, y_{iN})$.

Four criteria are used as the focus for the study: *salary, job location, job position* (type of work involved), and *prospects* (career development and promotion opportunities).

The following alternatives were used:

FIRM	SALARY	JOB LOCATION	POSITION	PROSPECTS
a1	$30 000	Very attractive	Good enough	Moderate
a2	$35 000	Unattractive	Almost ideal	Moderate
a3	$40 000	Adequate	Good enough	Almost none
a4	$35 000	Adequate	Not appropriate	Good
a5	$40 000	Unattractive	Good enough	Moderate

It is easy to note that in this case there are three possible values for each criterion. The greater the salary, the more attractive it would be to a rational subject. Thus, we have four criteria with three possible values each and the values upon each criterion are rank-ordered from the most to the least preferable one.

It is evident, that there are no dominated alternatives. Therefore, comparison of these alternatives requires some value function, which would take into account the advantages and disadvantages of each alternative upon each criterion.

Two decision support systems based on Multiattribute Utility Theory – MAUT[7], were used for the solution of the problem given above. These systems are LOGICAL DECISION [8] and DECAID [9]. The third DSS was ZAPROS[2,4] based on Verbal Decision Analysis.

Both decision support systems LOGICAL DECISION and DECAID were used to solve this task. Both systems implement ideas of multiattribute utility theory, providing possibilities for construction of an additive utility function for the case of risky decisions, and additive value function for decision making under certainty. In our study, we used only additive value functions.
The value function obtained from both systems would therefore have a linear form,

$$V(y_i) = \sum_{j=1}^{N} k_j V_j(y_{ij})$$

where: k_j is a coefficient of importance for the j-th criterion , y_{ij} is the value of alternative y_i on criterion j , and V_j is the value function for the j-th criterion.
Both systems are easy to use, have flexible dialogue and graphical tools to elicit DM preferences.

The main difference in the systems (besides interface) is the way of determination of numerical values upon separate criteria and criteria weights. In DECAID pure graphical (direct) estimation of alternatives is used (a point on the line of the size 1). In LOGICAL DECISION rather traditional for MAUT method of one criterion value function is used. To determine the parameters of this function the procedure of finding a sure thing for a lottery is used.

Criteria weights are also defined in a different manner in these two systems. In LOGICAL DECISION criteria weights are defined on the basis of trade-offs in a rather traditional way [7]. In DECAID weights are elicited directly (in a graphical way - point on a line), though the system provides also the possibility to make trade-offs, but after that the result is presented as points on lines. Thus, it is possible to consider it as direct elicitation of criteria weights.

Taking into account the commonness of the approach implemented in both systems and also the similarity of information, received from a DM in the process of task solution, an attempt to solve the above described task with the help of these systems must lead to the same result.

The third DSS is ZAPROS (see above). Only verbal measurements are used on all stages of this method. ZAPROS uses ranking rather than rating information, but the additive overall value rule is correct if there is an additive value function. In ZAPROS the additive rule does not provide the summation of values, but rather the means of obtaining pair-wise compensation between components of two alternatives.

The method also provides verification of the received comparisons for transitivity. The joint ordinal scale provides the possibility for the construction of partial ranking for every given set of alternatives.

Thus, this ranking may be used for comparison of initial 5 alternatives because in our task additive value function is supposed to be the right one and criteria were formed to be preferentially independent. This algorithm does not guarantee comparison of all alternatives because for some pairs of alternatives ZAPROS gives only incomparability relation.

Each subject from the group used all three DSS for the solution of the problem presented above. The difference in the outputs of methods consisted in following: some pairs of alternatives had not been compared with ZAPROS method. Simple method of preferences elicitation used by ZAPROS gives no possibility (in a general case) to compare all given alternatives; only partial ranking of alternatives is given.

In contrast to it, two other methods give the complete ranking for given alternatives. Also, LOGICAL DECISION and DECAID give numerical values of the utility for all alternatives.

The results of the experiment were analyzed in different form: the ranking of given alternatives, the ranking of special alternatives used in ZAPROS, the ranking of criteria weights and so on.

First of all, a very low correlation between the outputs of LOGICAL DECISION and DECAID was found. The ANOVA test demonstrated that for the group of subjects the outputs of LOGICAL DECISION and DECAID have not been statistically significant in measurements of criteria weights and ranking of alternatives.

The following results were very interesting: the outputs of pairs LOGICAL DECISION-ZAPROS and DECAID- ZAPROS were correlated and it was statistically significant. It means that only for alternatives compared by ZAPROS the relations were essentially the same.

It is possible to give the following explanation to the results.
The alternatives that could be ordered by ZAPROS are in the relations close to ordinal dominance. Such relations are more stable. More, they were constructed in a very reliable way: verbal measurement, a psychologically correct way of preference elicitation, possibility to check information and eliminate contradictions.

Two complete orders constructed by LOGICAL DECISION and DECAID were based on numerical measurements and weighted sum of alternatives estimations by criteria. The difference in the utility (even small) defined the final order of alternatives. The errors (even small) made by people while performing numerical measurements resulted in quite different orders of alternatives.

8 Quantitative and Qualitative Measurements

The majority of decision methods are of mathematical origin. The numbers are very convenient for any transformation. That is why, the presumption that people could feed numbers in decision methods and in computers is also very convenient for researchers.

The recent psychological findings on the limited capacity of human information processing system make rather doubtful the using of quantitative measurement in decision making methods. Many "natural" operations like nomination of criteria weights, comparison multicriteria alternatives are inclined to human biases and errors.

Verbal information is much more convenient for human communication. Computer can use verbal variables as the symbols. Logical transformation of symbols is possible, as well.

We regard decision making in the unstructured problems as the domain of the human activity where quantitative (the more so, objective) means of measurement are not developed, and it is unlikely that they will appear in future. Therefore, it is required to estimate the possibility of doing reliable qualitative measurements. Following R. Carnap [10], we turn to the methods of measuring physical magnitudes that were used before the advent of the reliable quantitative measurements. Before the invention of balances, for example, objects were compared in weight using two relationships — equivalence (E) and superiority (L), that is, people determined whether the objects are equal in weight or one is heavier than the other. There are three conditions to be satisfied by E and L [3]:

1. E and L must be mutually exclusive,
2. L is transitive, and
3. For two objects a and b are either a E b, or a L b, or b L a.

One can easily see that the above scheme enables one to carry out relatively simple comparisons of the objects in one quality (weight). It is required here that all objects be accessible to the measurement maker (expert).

Two more remarks are due. It is obvious that the thus-constructed absolute ordinal scale cannot have many values; otherwise, they will be poorly distinguishable by the measurement makers. To come to terms easier, it is required to identify commonly understandable and identically perceived points on the scale and explain their meaning in detail. Therefore, these scales must have detailed verbal definitions of the estimates (grades of quality). Moreover, these definitions focus on those estimates on the measurement scale that were emphasized by the persons constructing the scale (for example, they could be interested only in very heavy and very light objects). Thus, the estimates on the ordinal scale are defined both by the persons interested in one or another kind of measurement (in our case, it is the DM) and by the possibility of describing them verbally in a form understandable to the experts and the DM.

There is no reason to question the fact that before the coming of the reliable methods of quantitative measurement of the physical magnitudes, they were already measured qualitatively. Today, these methods could seem primitive because we have much more reliable quantitative methods. Yet, there is no doubt that the pre-quantitative (qualitative) methods of measuring physical magnitudes did exist. When they were superseded by the quantitative methods, they were treated with negligence as something 'unscientific' and obsolete. The progress of physics gave rise to the well-known statement that the science appears wherever the number (quantity) occurs. To our mind, these declarations refer mostly to the natural sciences, but in the sciences dealing with human behavior qualitative measurements were and will be the most reliable.

We could put the following requirements to human measurements in decision processes.

1. The measurements must be made in a language that is natural to DM and their environment.
2. In the case of quantitative variables (criteria) it is preferable to use discrete scales with the evaluations representing some intervals meaningful for "measurement makers".
3. In the case of qualitative measurements the ordinal scales with verbal evaluations are the best way of measurement.

Two more remarks are due. It is obvious that the thus-constructed ordinal scale cannot have many values; otherwise, they will be poorly distinguishable by "measurement makers". To come to terms easier, it is required to identify commonly understandable and identically perceived points on the scale and explain their meaning in detail. Therefore, these scales must have detailed verbal definitions of estimates (grades of quality). Moreover, these definitions focus on those estimates on the measurement scale that were emphasized by the persons constructing the scale (for example, they could be interested only in very hot and very cold objects). Thus, the estimates on the ordinal scale are defined both by the persons interested in one or another kind of measurement (in our case, it is the DM) and by the "distinguishability" of estimates, that is, the possibility of describing them verbally in a form understandable to experts and DM.

9 Conclusions

For the practical cases where an objective model of a problem is absent, the verbal description is a subjective model expressing DM subjective perception of reality. The tools of decision analysis are to be adapted to such description of a problem and to limited capacity of human information processing system.

Joint Ordinal Scale is a possible way to use verbal definitions of quality grades and logical transformation to compare multicriteria alternatives. In the cases with a large number of criteria and estimations on criteria scales, an analysis could be made in two stages: an approximate analysis on the basis of SJOS and detail analysis for some part of alternatives if it is needed.

In practical situations of project competition, the development of DM decision rule allows one to assign quality ranks to projects that directly correspond to DM preferences. It creates a basis for the implementation of a rational policy in important problems of the choice.

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